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OBSERVATIONS

MADE IN

SAVOY,

IN ORDER TO ASCERTAIN

THE HEIGHT OF MOUNTAINS

BY MEANS OF THE

BAROMETER;

BEING

An Examination of Mr. DE LUC's Rules, delivered in his *Recherches ſur les Modifications de l'Atmoſphere*.

MVSEVM BRITANNICVM

By Sir GEORGE SHUCKBURGH, Bart. F. R. S.

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MDCCLXXVII.

MVSEVM
BRITAN
NICVM

OBSERVATIONS, &c.

IN the courſe of my tour into Italy in the years 1775 and 1776, I made ſome ſtay at Geneva; which being in the neighbourhood of the Alps, and on that account a convenient home, induced me to make ſome obſervations upon thoſe mountains, which have been deſervedly objects of attention to the moſt incurious traveller. I was particularly deſirous of verifying the experiments with the barometer, in taking heights of different ſituations; a method that has been long known to the ingenious, though but rarely practiſed, and capable of but little preciſion till within theſe few years; and perhaps at preſent not ſo generally known as the convenience and utility of the method ſeems to require. I had provided myſelf with a conſiderable collection of inſtruments, or a kind of portable philoſophical cabinet, which I had had made expreſsly in London and Paris, in order to make ſuch experiments as might preſent themſelves to me *en courrant*; and which, either from want of acquaintance with the ſubject, want of time, or want of money, become rarely the object of travellers; but remain wholly unknown till princely munificence and philoſophic zeal (of which we have a recent inſtance) unite in producing them to the world. After the very celebrated and ingenious labours of Mr. DE LUC, farther inveſtigation of the ſubject of barometrical meaſurement might ſeem unneceſſary, if not invidious; but, furniſhed as I was with an apparatus every way ſufficient for the inquiry, finding myſelf in the country which had been the ſcene of his operations, and poſſeſſing ſome ſhare of his own zeal, I could not but gratify the curioſity I had to verify and repeat his experiments: if therefore in the purſuit of this inquiry I ſhould be led to a concluſion ſomething different from the reſult of his own obſervations, I am convinced that this diſtinguiſhed obſerver, of whoſe candour and talents I have an equal opinion, will impute it wholly to a love for truth; as with me the precept applies as ſtrongly to the philoſopher as to the hiſtorian, *Ne quid falſi audeat, ne quid veri non audeat dicere.*

But to proceed. The inſtruments I made uſe of in theſe operations were, two of RAMSDEN's barometers(a); three or four thermometers detached from the barometers, whoſe boiling and freezing points I had examined myſelf; an equatorial inſtrument, the circles of which were about ſeven inches diameter, made by RAMSDEN; a fifty-feet ſteel meaſuring chain; and three three-feet rods, two of deal and one of braſs, in order to examine and correct the chain, theſe latter made by BARADELLE at Paris. Beſides theſe I took with me a little bell-tent, which I found of great uſe, as it defended me from the wind and ſun; and I may remark, that the obſervations of the uppermoſt barometer were made in the tent.

My firſt ſeries of obſervations I propoſed to be on Mont Saleve(b), one of the Alps, ſituated about two leagues ſouth of Geneva, and preciſely on the ſame point where Mr. DE LUC had made his higheſt or fifteenth ſtation: this ſpot I learnt from his brother, whoſe civilities, both then and ſince, I ſhall frequently have occaſion to remember and mention.

(a) It may not be improper to remark, that the ſpecific gravity of the quickſilver of theſe barometers with 68° of heat was 13,61; the diameter of the bore of the tube 0,20 inch; and that of the reſervoir 1,5 inch.

(b) Mont Saleve extends near nine miles in length; is not quite 3300 feet in height above the Lake. That ſide of it which is next Geneva is for the moſt part a barren rock, the north-eaſt end of it being almoſt a perpendicular precipice; the other ſide of the mountain is leſs rude, of a more gentle acclivity, covered with trees, ſhrubs, and herbage, as is alſo the top, where is ſome of the fineſt paſture in the world. It is inhabited only by a few ſhepherds, who paſs the ſummer months here with their cattle, in little miſerable huts or barns: the remaining part of the year, *viz.* for about four or five months, it is covered with ſnow. This mountain contains chiefly a calcareous ſtone; and there is reaſon to believe that there is an iron ore in it, at leaſt in ſome parts of it, as a piece Mr. DE LUC, the brother, picked up near the ſouth-weſt end, I found, ſenſibly affected the magnet.

The place where I meaſured my baſe was in a field near the villages of Archamp and Neidens, not quite three miles in a horizontal line from the top of the rock whoſe height was to be determined (ſee the chart that accompanies this account). At the end of the baſe A I intended to place one of my barometers; and the other at the top of the rock, called the Pitton, at C; and with the above inſtruments meaſure the triangle ABC. The angles were taken both on the horary circle, which was brought parallel to the horizon, and alſo on the azimuth circle of the equatorial inſtrument; this made it, as it were, two different inſtruments independant of each other. The angles were moreover doubled, tripled, and quadrupled, on each arch; by this means the error of the center or axis of the inſtrument vaniſhed; the poſſible error in the diviſions, in the reading off, and in the coincidence of the wires in the teleſcope (which magnified forty times) with the ſignals placed at each angle of the triangle, was leſſened in proportion to the number of times the obſervation was repeated; and finally the mean of all was taken. The ſame was done with each angle at A, B, and C, horizontal as well as vertical, *viz.* the elevation of C above A and B was taken; and alſo the depreſſion of A and B below C. The advantage of this method was, that the error of the line of collimation, the effect of refraction, and of the curvature of the earth's ſurface, all became equal and contrary; by theſe means the little errors were diminiſhed, and great errors abſolutely avoided(c). I ſhall, however, beg leave to ſet down the operation at length reſpecting this one triangle, in order to ſhew the preciſion that may be expected from ſuch a geometrical proceſs; to remove the ſcruples of thoſe gentlemen who ſuſpect that accuracy is only to be obtained by large quadrants; and laſtly, to do juſtice and ſatisfaction to the celebrated artiſt who invented and made this valuable inſtrument.

(c) I muſt acknowledge here, that the attraction of the mountain creeps into the account uncorrected for, but only half of this quantity influences the mean reſult, as at the top it was nothing, and at the bottom of the mountain it could not exceed 10″ in the direction AC, as I find from a rough computation, the half of which = 5″ would give only four inches for the correction.

Determination of the Baſe.

	Ch.	Ft.	In.	Temper.
Length of the baſe AB (ſee the Chart) by the chain, firſt time, — — —	55	10	0	71°
Ditto, ſecond time, — — —	55	9	9¼	76
The mean, — —	55	9	10.87	73½

	Ft.	In.		
By frequent previous obſervations I determined (d) the length of the chain by compariſon with the braſs ſtandard rod reduced to 60° of heat, — —	50	0	0	60
Correction for 13½° of heat from expanſion, —	+ 0	0	05	
Diameter of the pins or arrows, one of which was uſed at each chain, and in ſuch manner, that this correction became always + — — —	+ 0	0	16	
Correct length of the chain as it was uſed in meaſuring the baſe, — — — —	50	0	21	
Multiply by the number of entire chains in the baſe, —			55	
	2750	11	55	
Add the parts of a chain, — —	+ 9	10	87	
True length of the baſe, as it was meaſured, —	2760	10	42	
Correction for the defect of level, taken with an inſtrument made on purpoſe, each time the chain was placed,	—	0	76 (e)	
The true horizontal diſtance between A and B becomes,	2760	9	66	

(d) It may be required, to what preciſion I could determine the length of my chain? I think certainly to within $\frac{1}{100}$ of an inch, or $\frac{1}{60000}$ of the whole length. The common GUNTER's chain of the ſhops is always ſubject to ſpring and ſtretch conſiderably; mine was made of hardened ſteel, on purpoſe to avoid this defect. It however ſtill preſerved ſome degree of elaſticity, for when pulled with a force of about ten pounds, it ſeemed = 0,12 inch longer than when laid gently on the floor without being ſtretched at all: the aſſumed length of the chain was ſuch as ſeemed to me probable from a moderate tenſion in common

Determination of the angles by the equatorial.

	On the azimuth circle.		On the equat. circle, the horary being converted into gradual diviſions.
∠ A by the 1ſt obſervation —	58° 27′ 30″	–	58° 28′ 30″
2d, — —	— 29 0	–	— 27 30
3d, — —	— 28 30	–	— 29 15
4th, — —	— 30 15	–	— 29 15
∠ taken four times over on the arch, —	233 54 15	–	233 54 30
The mean, —	58 28 49	–	58 28 37½

Laſtly, the mean of all from the two circles = 58° 28′ 43¾″ = ∠ at A.

∠ B by the 1ſt obſervation, —	111 54 45	–	111 53 0
2d, — —	— 51 30	–	— 52 30
3d, — —	— 50 30	–	— 50 45
∠ taken three times over on the arch, —	335 36 45	–	335 36 15
Mean, — —	111 52 15	–	111 52 5

Mean of all from the two circles = 111° 52′ 10″ = ∠ at B.

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common uſing it. It may perhaps not be out of place to remark here, that the rods with which the chain was examined, agreed exactly with the ſcales of the barometers; at leaſt the difference in nine inches, taken in different parts of the ſcale, did not appear to exceed $\frac{1}{1000}$ of an inch.

(i) The precaution in taking the inclination of the chain every time, if the baſe be nearly a plain, as is the caſe in many meadows, ſeems to be unneceſſary; for this ſame correction, deduced from the inclination of the baſe obſerved at A and B, comes out —0,99 inch, only 0,23 inch different, a quantity wholly inconſiderable.

		On the azimuth circle.				On the equat. circle.		
		°	′	″		°	′	″
∠ c by the 1ſt obſervation,	—	9	39	0	-	9	38	30
2d,	— —	-	39	0	-	-	38	15
3d,	— —	-	38	45	-	-	39	45
∠ taken four times over on the arch,	—	38	35	45	-	38	34	45
Mean,	— —	9	38	$56\frac{1}{4}$	-	9	38	$41\frac{1}{4}$

Mean of the two circles, = 9° 38′ $48\frac{3}{4}$″ = ∠ at c.

		By actual obſervation.				Angles finally corrected.		
		°	′	″		°	′	″
∠ at A,	— —	58	28	$43\frac{1}{4}$	These angles corrected by adding 6″ to each (the ſum of their errors, or defect from 180° being —18″) become,	58	28	$49\frac{1}{4}$
∠ at B,	— —	111	52	10		111	52	16
∠ at C,	— —	9	38	$48\frac{3}{4}$		9	38	$54\frac{3}{4}$
Sum of the three angles	=	179	59	42	Sum,	180	0	0
Taken from		180	0	0				
Leaves the difference = ſum of the errors,			—	18				

It is highly curious and ſatisfactory to ſee the amazing correſpondency of theſe obſervations, made with an inſtrument of only $3\frac{1}{2}$ inches radius, whereon an angle of one minute is about equal $\frac{1}{1300}$ inch; and I think we may fairly conclude, that the corrected mean reſult of theſe obſervations is true to within 6″ or 8″(f); which, as may be proved hereafter, would occaſion an error of only three feet in the diſtance of the mountains, and ſeven inches in the height. I proceed next to the vertical angles.

(f) I may have a future occaſion to ſpeak of the accuracy of this inſtrument for aſtronomical purpoſes; but I cannot omit this opportunity of mentioning one, *viz.* in taking the latitude of the city of Amiens in Picardy, where I had thirteen obſervations by the ſtars and Sun, the mean of which differed 25″ from the extremes, and only 3″ from the reſult of Mr. CASSINI's obſervations, made, I believe, with a nine-feet zenith ſector, as related in *La Meridienne de Paris verifiée.*

Determination of the inclination of the ſides AC, BC, *and* AB, *with the horizon; the height of the eye at the inſtrument being four feet above the ground.*

Altitude from below at A.	°	′	″
Inclination of AC, —	10	33	2
Correction for the part of the ſignal which was obſerved,	—	1	38
Correction for the line of collimation, —	—	0	59
Correct for the refraction,	—	0	27
True Altitude of C from A,	10	29	58

Depreſſion from above at C.	°	′	″
	10	29	18
Correct for the ſignal, —		+	16
——— for the line of collimation, —		+	59
——— for refraction,		+	27
True depreſſion of A from C,	10	31	0
Arch intercepted between, or curvature, —		— 2	30
True altitude of C from A deduced from the obſervation at C,	10	28	30

Mean corrected altitude of C from A = 10° 29′ 14″[g].

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[g] If the computation were to be made from either of the obſervations taken ſeparately, the difference would amount to only three feet in the height of C; and this may either be in the correction of the line of collimation, the effect of refraction, or in miſtaking the part of the ſignal that was obſerved: for, whilſt I was gone to the top of the mountain, ſome peaſants poſſeſſed themſelves of the handkerchiefs I had fixed to the ſignals below in order to have a conſpicuous and determined point.

Altitude from below at B.	°	′	″
Inclination of BC, —	11	20	26
Correct for the part of the ſignal obſerved, —	—	1	38
Error of collimation,	—	0	59
Correct for refraction,	—	0	26
True altitude of C from B,	11	17	23

Depreſſion from above at C.	°	′	″
	11	19	47
Correct for the ſignal, —		—	59
Error of collimation, —		+	59
Effect of refraction, —		+	26
True depreſſion of B from C,	11	20	18
Arch intercepted, or curvature, —	—	2	18
True altitude of C from B, deduced from the obſervation at C, —	11	18	0

Mean of the two, or corrected altitude of C from B $= 11° 17′ 41\frac{1}{2}″$.

Altitude at A.	°	′	″
∠ of inclination of AB the baſe, —	0	27	0
Error of the line of collimation, —	—	0	59
Correct altitude of B from A,	0	26	1

Depreſſion at B.	°	′	″
	0	27	4
Error of collimation,	+	0	59
Correct depreſſion of A from B, —	0	28	3
Arch intercepted, —	—	0	27
Altitude of B from A deduced from the obſervation at B, —	0	27	36

Mean of the two, or corrected altitude of B from A $= 0° 26′ 49″$(b).

(b) It ſhould ſeem from theſe two obſervations, that the error of the line of collimation had been aſſumed too great; it has however, as I have before obſerved, nothing to do with the mean reſult: and this is, perhaps, one of the beſt means of diſcovering the error of collimation, and the very method Mr. DE LUC uſed, to adjuſt his level, though, as I have been informed by his brother, without taking into the account the effect of curvature, which, if his horizontal marks were 2000 feet diſtant from each other, would amount to 20″, and the error to half that quantity.

I have thus, in a manner rather prolix, given a detail of the methods uſed to aſcertain the quantity of the different angles. It may be of uſe on a like occaſion, and will at leaſt ſerve to determine within what limits the error of the final reſult may be expected to lye, as on the preciſion of the geometrical operations all the compariſons of the barometrical ones depend. This proceſs once mentioned will exempt me and the reader from the trouble a ſecond time, when he is informed, that the ſame fidelity and pains were employed (where the circumſtances would admit) in all the trigonometrical obſervations, of which the annexed chart is a ſummary. I proceed now to the determination of the ſides, the computations of which are too well known to enter into this paper.

	Feet.
Side AB	2760.8
AC	15286.4
BC	14041.7

	Feet.
Theſe with the angles give for the height of C above A,	2835 07
The height of C above B,	2806.27
The height of B above A,	22.18
Theſe two added give the height of C above A, deduced from the obſervation at B,	2828.45
But the height by actual obſervation at A was,	2835.07
Then the mean of the two,	2831.76

which is probably within three or four feet of the truth, or about one foot in a thouſand.

Having thus the perpendicular height, as I think, very accurately aſcertained, it remained for me to take the altitude of the barometer at each ſtation A and C, and if poſſible with equal preciſion. Theſe obſervations it would be too tedious to ſet down at length. I ſhall, however, premiſe the following particulars. Every obſervation of the barometer was triple; that is, the height was read off three different times, and the mean taken; but from once reading only I could be ſure of the height to $\frac{2}{1000}$ of an inch, excluſive of the error of the diviſions, which in ſome places might amount to that quantity; this the nonius would itſelf diſcover and even correct by eſtimation. At every ſeries of obſervations the float at the bottom was readjuſted, ſo that I could conſtantly be ſure of an alteration of the weight of the atmoſphere expreſſed by 0.002 inch of quickſilver, if not of half that quantity. Finally, the difference of the two barometers (*i*) was conſtantly taken, after being left three-quarters

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(*i*) It may be concluded, that this difference ſhould be conſtant, and always the ſame; but, from what cauſe I know not, it did not appear ſo to me. In my journal for the weather for 1775, I find the following note: from a mean of ſeventeen obſervations between Auguſt 12th and Sept. 1. *viz.* before, at, and after, my expedition to Mont Saleve and the Mole, I find the difference between my two barometers =,0042 inch, N° 1. ſtanding the higheſt; in theſe compariſons, however, the extremes ſometimes differed from the mean =,006. And in my paſſage over Mont Cenis, Dec. 1. barometer N° 1. ſtood lower than N° 2. by ters of an hour or more in the ſame place, to acquire the true temperature of the air, and this before and after every expedition. The fractional parts of a degree on both the attached and detached thermometers were noted only by eſtimation, but written down to 10ths, being more convenient in the computation; for I may remark, that one-third of a degree on the attached thermometer is equal to about $\frac{1}{1000}$ inch on the barometer; this attention, therefore, to the ſub-diviſions of the degrees became neceſſary. I conclude, laſtly, with preſuming, that the weight (k) of any column of air may be meaſured with theſe barometers to ,008 inch, though all the errors ſhould lye the ſame way.

Leaving Geneva about half paſt ſix in the morning, Auguſt 20th, I arrived at the place A of my baſe a little before eight; near to which there happened to be a ſhepherd's houſe, in which I left one of my barometers (N° 1.) with a ſervant, to examine and obſerve it every five or ten minutes for near nine hours ſucceſſively,

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by —,013 inch: it is difficult to account for this. May 10th, 1776, at Rome, N° 1. ſtood loweſt by —,001. June 12th, at Naples, N° 1. ſtood loweſt by —,008. Sept. 10th, in London, N° 1. ſtood higheſt by +.006. Theſe apparent variations may poſſibly ariſe from ſome alteration in the frame-work of the barometers through moiſture, &c.

(*k*) I muſt not be underſtood to mean, that the length of any column of air may be meaſured to an equal accuracy, even though our theory ſhould be perfect: this will be the ſubject of inquiry in its proper place.

until I returned; the windows and doors of the room, in which the inſtrument was placed, being left open, by which means there was a free communication with the outward air, and the barometer not expoſed to the Sun. The detached thermometer was hung on the window towards the north-eaſt, where there was neither direct nor reflected heat from the Sun (l). The two barometers

(l) I have thought proper to mention this, as it is almoſt the only circumſtance wherein my method of obſerving differed from Mr. DE LUC's, whoſe thermometers (if I miſtake not) were hung always in the Sun, and probably for this reaſon, becauſe the column of the atmoſphere between the two barometers, whoſe mean heat is to be determined, is (if the Snn ſhine) all expoſed to the Sun. I have, however, always preferred hanging them in the ſhade, and I give the following reaſons: all ſpurious and local heat from reflection is more eaſily avoided; no concentrated and falſe heat is acquired by the mounting, and thence communicated to the tube, even though the ball ſhould be inſulated; and, finally, becauſe I ſuſpect the real temperature of the atmoſphere in the Sun and in the ſhade to be the ſame, or at leaſt inſenſibly different. This may be thought to be advancing too much; but, to be ſatisfied of the poſition, I made no leſs than four-ſcore obſervations with four different thermometers of very different mounting, hung alternately expoſed to the Sun's rays, and ſcreened from them by the ſhade of a tree, in an open plain at ſome diſtance from the town of Geneva. The reſult was, that my beſt thermometer, with the ball inſulated, differed only 2° in the different ſituations; the others, more or leſs, as they were more or leſs connected with the frames in which they hung. One of them, incloſed in a glaſs tube, roſe 12° higher than the true temperature, which was 77°. It ſhould ſeem then, that the variety in the mounting occaſioned this difference; and this effect of the materials, of which the inſtrument is made, cannot be wholly avoided, as the glaſs itſelf, which conſtitutes the ball of the thermometer, will acquire and contain more or leſs, in proportion to its thickneſs and opacity. If a thermometer were perfect, it would reflect all the rays that it receives. More might be added to corroborate this idea, but it would ſwell this note to an unwarrantable length.

were here compared; and at a quarter after nine, beginning my walk, I arrived, not without ſome fatigue, at the top of the mountain about noon. The view from thence was incredibly beautiful. Every object, that from Geneva was ſtriking, from thence appeared with an additional effect. The mountains ſeemed higher and nearer; the plain appeared a more perfect level, the ſmall inequalities from this height becoming inſenſible; and a larger portion of the lake preſented itſelf: behind me an innumerable collection of naked points and precipices, all new objects, that from below are hid by the mountain, afforded freſh and moſt aſtoniſhing ideas of this very ſingular part of the creation. The clouds however (for it was a little hazy) unfortunately prevented my ſeeing Mont Blanc and the Glacieres, which were ſtill farther behind. Some of the clouds were below me, and very near; exhibiting to me, at that time, a very ſingular phenomenon of the thunder grumbling under my feet. I was occupied here between four and five hours with different obſervations. The barometrical ones I am now going to relate; and I ſhall at the ſame time give the computations of them according to Mr. DE LUC's method, or rather according to Dr. HORSLEY's reduction of it to the ſcales and meaſures of this country (*vide* Philoſ. Tranſ. vol. LXIV.) with this difference, that I have reckoned the equation for the expanſion of quickſilver =,00323 inch for every degree of FAHRENHEIT'S thermometer in a column of 30 inches, inſtead of ,00312 which Mr. DE LUC uſed; the former I had collected from ſome of my own experiments made at Oxford in the beginning of the year 1773: this difference will not, however, occaſion an alteration in the reſult of any one of my obſervations of more than five inches, and may therefore be conſidered as of no account. Of the real value of this correction I ſhall ſpeak more hereafter.

The barometer was ſet up on the mountain at one o'clock, and left an hour and a quarter to acquire the temperature of the tent in which it was placed, before the firſt regular ſeries of obſervation was taken. The ſucceeding obſervations were made at intervals of near an hour each. I have ventured to ſet down the height of the barometer to ,0001 inch; but this is only the mean from three or four readings off. It ſeems that the heat of the tent was conſiderably greater than that of the external air; this, however, can only influence the expanſion of the quickſilver, ſhewn by the attached thermometer, and not the preſſure of the atmoſphere. Laſtly, the true difference in the height of the reſervoirs of the two barometers, by compariſon with A and C, was found equal 2831.3 feet geometrically.

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Compariſon of the firſt ſeries.

Obſervations at the top of the mountain at c.

	Barom. N° 2. above at c. In. Pts.		Therm. attached.	Therm. detached.
	25.7120		78°.0	65°.0
Correct for the diff. of the 2 attached therm. 5°.9,	— 162			
Barometer at the top,	25.6958	Log.	4098621	
——— below,	28.3951	Log.	4532434	
Difference, or fall of the quickſilver,	2.6993	Diff. of Log.	433.813	the height in Engliſh fathoms,
Correct for 29°.7 of heat,	—	—	+ 28.728	
Correct height in fathom,	—	—	462.541	
			× 6	
Height in Engliſh feet by the barometer,		—	2775.246	
Height by the trig. method,	—	—	2831.3	
Difference, or error of the barometer $\frac{108}{10000}$,			— 56.1	

Obſervations below at A.

	Barom. N° 1. below at A. In. Pts.	Therm. attached.	Therm. detached.	
	28.3990	72°.1	73°.9	
Correct for the diff. of the barometer,	— 39		65.0	heat at c.
			69.4	mean heat of the air.
	28.3951		39.7	ſtand. temp. according to Dr. HORSLEY.
			+ 29.7	difference.

A detached thermometer in the tent ſtood at 72°.

Compariſon of the ſecond Series.

Obſervation at the top of the mount at c.

	Barom. N° 2. above at c. In. Pts.	Therm. attached.	Therm. detached.	
	27.7025	73.4°	64.0°	
Correct for the Diff. of the two attached therm.	— 50			
Barometer at the top,	25.6975	Log.	4098908	
——— below, —	28.3901	Log.	4531669	
Difference, or fall of the quickſilver, —	2.6926	Diff. of Log.	432.751	approx. height in English fathoms.
Correct for 28°.8 of heat,	—	—	+ 27.787	
Corrected height in fathoms,	—	—	460.538 × 6	
Height in feet by the barometer		—	2763.228	
——— by the trig. method,		—	2831.3	
Difference, or error of the barometer $\frac{240}{100000}$,			— 68.1	

Obſervation below at A.

	Barom. N° 1. below at A. In. Pts.	Therm. attached.	Therm. detached.	
	28.3940	71.6°	73.0°	
Correct for the diff. of barometer,	— 39		64.0	heat at c.
			68.5	mean heat.
	28.3901		39.7	ſtandard temperature.
			+ 28.8	difference.

A detached thermometer in the tent ſtood at 69°.

During theſe obſervations the wind was S.W.; the weather hazy, accompanied with a little thunder.

Com-

Compariſon of the third ſeries.

Obſervations at the top near c.

	Barom. N° 2. above at c. In. Pts.	Therm. attached.	Therm. detached.	
	25.6900	69°.7	62°.0	
Correct for the diff. of the 2 attached therm. 1°.4,	+ 38			
Barometer at the top,	25.6938	Log.	4098283	
——— below, —	28.3896	Log.	4531593	
Difference, or fall of the quickſilver, —	2.6958	Diff. of Log.	433.310	Approx. height in fathoms.
Correct for 27°.5 of heat,	—	—	+ 26.582	
Correct height in fathoms,	—	—	459.892 × 6	
Height in feet by the barometer,		—	2759.352	
——— by the trig. method,	—	—	2831.3	
Difference, or error of the barometer $\frac{254}{10000}$,			— 71.9	

Obſervations below near to A.

	Barom. N° 1. below at A. In. Pts.	Therm. attached.	Therm. detached.	
	28.3935	71°.1.	72°.5	
Correct for the diff. of barometer,	— 39		62.0	heat at c.
	28.3896		67.2	mean heat.
			39.7	ſtandard temperature.
			+ 27.5	difference.

A detached thermometer in the tent ſtood at 65°.

Theſe obſervations then ſeem to prove that the barometrical rules were a little defective as to the true ratio between the gravities of air and quickſilver, *viz.* in the value of an inch of quickſilver in the torricellian tube, expreſſed in inches of the atmoſphere with a given temperature. The firſt compariſon gives for this error in defect −19.8 feet in every 1000 feet; the ſecond, 24.0 feet; and the laſt, 25.4 feet: the mean of the three is 23.1 feet; and by ſo much we may conclude that theſe rules, in greater heights alſo, will give the difference of elevation too little, *viz.* by $\frac{1}{43}$ nearly (m). But it will be fair to make the experiment.

(m) Leſt any ſuſpicion ſhould ariſe of a diſagreement between the actual meaſures taken by Mr. DE LUC and myſelf, I may obſerve, that the mean reſult of three obſervations, which I made independently of each other on the height of the Pitton or point C above the lake of Geneva, agree with the mean reſult of Mr. DE LUC's operation from the levelling and the quadrant, to leſs than twelve inches; a greater correſpondency than which cannot be expected: and this was the true reaſon why I choſe the ſame ſpot he had pitched upon. "*Le rocher* "*iſolé, qui domine toute la montagne.*" As a further confirmation, I compared his ſtandard ſteel rod of twelve Paris inches, which his brother obligingly furniſhed me with, with my braſs one, and found twelve inches on Mr. DE LUC's rule

was on my rule, with 71° of heat, — —	12.784 Eng. inches.
Correction for the difference of expanſion between braſs and ſteel with 16° of heat, — —	+ 07
Length of Mr. DE LUC's French foot with 55°, —	12.7847
True length of the French foot (*vide* Phil. Tranſ.)	12.7890
Error or difference from the true Paris foot —	—,0043 = $\frac{1}{3000}$ nearly.

The Mole is a convenient, inſulated mountain, ſituated about eighteen miles eaſt of Geneva, and riſing near five thouſand feet above the lake, much higher than any body, that I know of, has ever made theſe experiments at, with the required preciſion. On this ſummit I determined to confirm or correct my diſcovery, and communicated my intentions to Mr. DE SAUSSURE, a very ingenious gentleman of this place, and well ſkilled in various parts of natural and experimental philoſophy, who gave me all the information neceſſary, and obligingly promiſed to accompany me, as did alſo Mr. TREMBLEY, aſſiſtant to Mr. MALLET, well known in the aſtronomical world. This expedition was undertaken in the latter end of Auguſt and beginning of September. I ſhall here beg leave to ſet the reader down at the bottom of the mountain, and flatter myſelf he will accompany me to the top. It was about five in the afternoon when we left St. Joire, a wretched little village at the foot of the mountain to the eaſt, and where we had dined in a moſt miſerable *auberge*, preparing to aſcend the ſummit on foot, being ſeven or eight in company, including guides and ſervants, who carried my inſtruments, proviſions, &c.; the former conſiſting of the equatorial, the barometer, different thermometers, electrical balls, an hygrometer, and a dipping-needle; together with another barometer of Mr. DE LUC's conſtruction, a variation-needle, a level belonging to Mr. DE SAUSSURE, and a tent. Thus accoutered we proceeded up an aſcent, not however very ſteep, for three hours and a half without intermiſſion, the path leading in a ſpiral kind of direction, very rugged and full of looſe pieces of rock that are brought down with the melting ſnows, paſſing through romantic woods of fine firs and other trees, interſperſed here and there with a thin ſoil of excellent paſture. Before we arrived at the hut, where we were to ſleep (for our intention was to lay upon the mountain that night, in order to have the more time the next morning for our operations) having walked on a little too far before, we loſt ſight of our guides. We called ſeveral times, but were never anſwered:—the night was now coming on; a kind of fog appeared, with ſmall rain; our ſituation became ſomewhat embarraſſing. We called again, but were anſwered by nothing but an echo, the place being a moſt profound ſolitude. We began now to conſider ourſelves as loſt. Mr. DE SAUSSURE, though he had been ſeven or eight times before upon the mountain, found himſelf in doubt concerning the way; but after a ſhort dilemma thought it beſt to proceed. We did; and now began to perceive at a diſtance ſome little huts or hovels indiſtinctly: a few more ſteps aſſured us we were right, and about nine o'clock we had the good luck to find ourſelves at the very hovel, where we were to reſt that night. I own I now found myſelf quite contented, though I did not at all know what kind of place I was going to enter. It proved to be a little hut made of boards, conſiſting of one apartment only, eighteen or twenty feet ſquare, and about twelve high in the center, without any windows or chimney for the ſmoke, except what was made by the holes in the roof, and the interſtices between the boards at the ſides, which were rudely put together, ſcarce cloſer than park-palings, affording an equal entrance to the wind, rain, and ſnow; for as theſe hovels are inhabited only for about four months in the ſummer, they are conſtructed without the leaſt mortar or cement in the world; an humiliating witneſs this, how ſimple the architecture which nature and neceſſity ſuggeſt. On entering we found a comfortable fire, and the little *cabane* inhabited by a couple of Alpine ſhepherdeſſes and their two cows, on whoſe whey and ſome very coarſe bread they wholly ſubſiſted, not diſcontented but even proud of their lot; and who, out of a ſingular ſpecies of contempt, call the inhabitants of the plain *mange-rotis*, that is, eaters of roaſt-meat. Their language too was different; not French nor Italian, but partaking ſomething of both; or, as I have been ſince informed, a corruption of the ancient Celtic.

A few minutes after our arrival our guides rejoined us: it was now night, and in this rather too artleſs habitation we were obliged to lay in a little loft over the cows, our beds ſome leaves and clean hay, and my bolſter my portmanteau(n). I had had the caution to bring ſome ſheets with me, and, being a little tired with my walking, ſlept five hours pretty ſoundly, though much ſtarved, having no other curtains than what this wooden canopy afforded, through which the ſtars ſhone moſt brilliantly. Between four and five we aroſe; found the heavens beautifully ſerene, and, having eaten ſome of our proviſions, left this habitation, which might be ſituated about two-thirds of the way up the mountain; and beginning our march about half after five reached the ſummit a quarter before ſeven; but not without a good deal of climbing, and ſometimes up an aſcent of near 40° for ſeveral hundred feet. One of my ſervants, before he got half way, found his head turn round, and himſelf ſo giddy, at the height and precipices (a frequent effect in theſe ſort of places) that he was obliged to return to the hut. In the aſcent I ſaw the Sun riſing behind one of the neighbouring alps with a moſt beautiful effect, and the ſhadow of the mountain we were then upon extended fifteen or twenty miles weſt. We had now reached the ſummit; and there my curioſity finiſhed in aſtoniſhment. I perceived myſelf elevated 6000 feet in the atmoſphere, and ſtanding as it were on a knife-edge, for ſuch is the figure of the ridge or top of this mountain; length without breadth, or the leaſt appearance of a plain, as I had expected to find. Before me an immediate precipice, *à pic*, of above 1000 feet, and behind me the very ſteep aſcent I had juſt now mounted. I was imprudently the firſt of the company: the ſurprize was perfect horror, and two ſteps further had ſent me headlong from the rock.

(n) ———————— Frigida parvas
Præberet ſpelunca domos, ignemque, laremque,
Et pecus, et dominos communi clauderet umbrâ;
Sylveſtrem montana torum cùm ſterneret uxor
Frondibus et culmo. JUV. Sat. vi.

On this ſpot, with ſome difficulty, we fixed the inſtruments, and commenced our operations, after ſome time ſpent in admiration at the proſpect, and familiarizing myſelf to the ſcene. Before me, at ſome diſtance, was ſpread the plain in which lay Geneva and the lake; behind it roſe the Dole, and the long chain of Mont Jura as far as the fort La Cluſe, which we entirely commanded, as well as ſome of the country beyond it. A little to the left, and much nearer, lay Mont Saleve, which from this height appeared an inconſiderable hill: to the right and left nothing but immenſe mountains, and pointed rocks of every poſſible ſhape, and forming tremendous precipices. In the vale beneath, ſeveral little hamlets, and the moſt beautiful paſturages, with the river Arve winding and ſoftening the ſcene; from whence aroſe a thick evaporation, collecting itſelf into clouds, which on the lake, that was quite covered with them, had the appearance of a ſea of cotton, the Sun-beams playing in the upper ſurface of them with thoſe tints that are ſeen in a fine evening. To the ſouth-weſt appeared the lake of Annecy; behind us, taking up one-fifth of our horizon, lay the Glacieres, and amongſt them, towering above all the reſt, ſtood Mont Blanc. The circumference of the horizon might be about 200 Engliſh miles; and, though not one of the moſt extenſive, yet certainly one of the moſt varied in the world. From this ſpot the clouds had a ſtriking appearance to an inhabitant of the plain; very few of them at above one-fifth of the height that we were now at; not governed by the wind, but moving in every poſſible direction; ſome of them ſeemed creeping along the ground, whilſt others were riſing perpendicularly between the hills. And I may here remark, that from Geneva I have obſerved the clouds were generally three days in the week below the ſummit of Mont Saleve; ſo that the ordinary region of theſe vapours ſeems to be at that height in the atmoſphere, where the barometer would ſtand at about 26 inches in this climate.

While at the top of the Mole, I was very ſenſible of the cold, there being a briſk wind, which, though ſouth, came over the mountains of ice, and was very keen; inſomuch that, about two hours after I had been there, I nearly loſt the uſe of my fingers, and found my lips much affected and parched from the tranſition, having been a good deal heated in aſcending with two waiſtcoats and a great coat on. The thermometer, however, when I firſt mounted, ſtood no lower than 48°. I muſt here aſk pardon for this long digreſſion, which I have ventured to tranſcribe from my journal written upon the ſpot.

To return then to the obſervations. After what has been ſaid reſpecting thoſe on Mont Saleve, it will ſuffice here to mention, that by repeated meaſurements I determined the horizontal length of the baſe 1, 2 (ſee the chart) to be $=1250$ ft. 3.9 inch; the $\angle$ at $1=95^\circ\, 37'\, 28''$; $\angle$ at $2=77^\circ\, 48'\, 53''$; and the $\angle$ at $3=6^\circ\, 33'\, 49''$. The mean corrected angle of elevation of 3 from 1 $=21^\circ\, 29'\, 34''$; ditto of 3 from $2=21^\circ\, 3'\, 41''$; and laſtly, the elevation of 2 from $1=0^\circ\, 47'\, 24''$.

	Feet.
Theſe obſervations give for the length of the ſide 1, 3, —	10691.9
— — — — 2, 3, —	10886.7
Height of 3 above 1, — — —	4212.8
——— 3 above 2, — — — —	4194.8
——— 2 above 1, — — — —	17.2
And conſequently, 3 above 1 deduced from the obſervation at 2, —	4212.0
And laſtly, the mean height of 3 above 1 from the determination at each end of the baſe, — — —	4212.4

The difference in height, however, between the two barometers was only 4211.3 feet.

Here follow the barometrical obſervations (o), and their reduction.

(o) Made between the hours of eight and twelve, in the open air and not in the tent, which could not be pitched on accent of the ſmallneſs of the plain at the ſummit; a briſk ſouth wind, but fair. The barometer was ſcreened by an umbrella.

Com-

Compariſon of the firſt ſeries on the Mole.

Obſervation at the top at 3.

	Barom. N° 2. above at c.	Therm. attached.	Therm. detached.	
	In. Pts.	°	°	
	24.1437	57.0	54.8	
Correct for the Diff. of the two attached therm. 3°.4,	+ 88			
Barometer at the top,	24.1525	Log.	3829621	
——— below, —	28.1253	Log.	4490971	
Difference, or fall of the quickſilver, —	3.9728	Diff. of Log.	661.350	approx. height in fathoms.
Correct for 18°.6 of heat,	—	—	+ 27.431	
Corrected height in fathoms,	—	—	688.781	
			× 6	
Height in feet by the barometer		—	4132.686	
——— by the geometrical meaſurement,			4211.3.	
Difference, or error of the barometer,			— 78.6 = $\frac{187}{10000}$.	

Obſervation below at 1.

	Barom. N° 1. below at 1.	Therm. attached.	Therm. detached.
	In. Pts.	°	°
	28.1295	60.4	61.9
Correct for the diff. of barometer,	— 42		54.8 heat at 3.
			58.3 mean heat.
	28.1253		39.7 ſtandard temperature.
			+ 18.6 difference.

Com-

Compariſon of the ſecond Series.

Obſervation at the top at 3.

	Barom. N° 2. above at 3. In. Pts.	Therm. attached.	Therm. detached.
	24.1420	56°.9	56°.0
Correct for the diff. of the two attached therm. 3°.5,	+ 91		
	24.1511	Log. 3829369	
	28.1258	Log. 4491049	
Difference, or fall of the quickſilver, —	3.9747	Diff. of Log. 661.680	approx. height in fathoms.
Correct for 19°.2 of heat,	— —	+ 28.330	
Correct height in fathoms,	— —	690.010	
		× 6	
Height in feet by the barometer,	—	4140.06	
——— by the geom. method,	—	4211.3	
Difference, or error of the barometer,		— 71.2 = $\frac{169}{10000}$.	

Obſervation below at 1.

	Barom. N° 1. below at 1. In. Pts.	Therm. attached.	Therm. detached.
	28.1300	60°.4	61°.8
Correct for the diff. of barometer,	— 42		56.0 heat at 3.
			58.9 mean heat.
	28.1258		39.7 ſtandard temperature.
			— 19.2 difference.

Compariſon of the third Series.

Obſervation at the top at 3.

	Barom. N° 2. above at 3. In. Pts.	Therm. attached.	Therm. detached.	
	24.1670	56°.0	56°.0	
Correct for the diff. of the two attached therm. 4°9.	+ 127		⊙ 57.5 (p)	
	24.1797	Log.	3834509	
	28.1278	Log.	4491358	
Difference, or fall of the quickſilver,	3.9481	Diff. of Log.	656.849	Approx. height in fathoms.
Correct for 19°8 of heat,	—	—	+ 29.0	
Correct height in fathoms,	—	—	685.849 × 6	
Height in feet by the barometer,		—	4115.094	
——— by the geom. method,		—	4211.5	
Difference, or error of the barometer,			— 96.2 = $\frac{228}{10000}$.	

Obſervation below at 1.

	Barom. N° 2. below at 2. In. Pts.	Therm. attached.	Therm. detached.
	28.1320	60°.9	63°.0
Correct for the diff. of the barometer,	— 42		56.0 heat at 3.
	28.1278		59.5 mean heat.
			39.7 ſtandard temperature.
			19.8 difference.

(*p*) In this column for the detached thermometer at the top of the mountain, in this and the following obſervations, are inſerted two numbers; the upper one expreſſing the heat in the ſhade; and the lower one, with this mark ⊙ prefixed, the heat in the Sun. The computation, however, is made from the former; this may ſerve to ſhew the difference.

Com-

Compariſon of the fourth ſeries.

Obſervation at the top at 3.

	Barom. N° 2. above at 3. In. Pts.	Therm. attached.	Therm. detached.	
	24.1780	57°.2	56°.0 ⊙ 57.5	
Correct for the diff. of the two attached therm. 4°6,	+ 119			
	24.1899	Log.	3836341	
	28.1318	Log.	4491976	
Difference, or fall of the quickſilver,	3.9419	Diff. of Log.	655.635	approx. height in fathoms.
Correct for 20°.3 of heat,	—	—	+ 29 678	
Correct height in fathoms,	—	—	685.313 × 6	
Height in feet by the barometer,		—	4111.878	
——— by the geom. method,		—	4211.3	
Difference, or error of the barometer,			— 99.4 = $\frac{236}{10000}$.	

Obſervation below at 1.

	Barom. N° 1. below at 1. In. Pts.	Therm. attached.	Therm. detached.
	28.1360	61°.8	63°.9
Correct for the diff. of the barometer,	— 42		56.0 heat at 3.
			60.0 mean heat.
	28.1318		39.7 ſtandard temperature.
			+ 20.3 difference.

Com-

Compariſon of the fifth ſeries.

Obſervations at the top at 3.

	Barom. Nº 2. above at 3. In. Pts.	Therm. attached.	Therm. detached.	
	24.1840	59°.6	57°.0	
Correct for the diff. of the 2 attached therm. 2°.8,	+ 73		59.3	
	24.1913	Log.	3836592	
	28.1308	Log.	4491820	
Difference, or fall of the quickſilver,	3.9395	Diff. of Log.	655.228	approx. height in fathoms,
Correct for 20°.8 of heat,	—	—	+ 30.391	
Correct height in fathom,	—	—	686.619	
			× 6	
Height in feet by the barometer,		—	4113.714	
———— by the geom. method,		—	4211.3	
Difference, or error of the barometer,			— 97.6 = $\frac{231}{10000}$.	

Obſervations below at 1.

	Barom. Nº 1. below at 1. In. Pts.	Therm. attached.	Therm. detached.
	28.1350	62°.4	64°.0
Correct for the diff. of the barometer,	— 42		57.0 heat at 3.
	28.1308		60.3 mean heat.
			39.7 ſtandard temperature.
			+ 20.8 difference.

Compariſon of the ſixth ſeries.

Obſervation at the top at 3.

	Barom. N° 2. above at 3. In. Pts.		Therm. attached.	Therm. detached.
	24.1900		61°.0	57°.0
Correct for the diff. of the two attached therm. 1°6,	+ 41			⊙ 60.0
	24.1941	Log.	3837095	
	28.1268	Log.	4491204	
Difference, or fall of the quickſilver,	3.9327	Diff. of Log.	654.157	approx. height in fathoms.
Correction for 20°6 of heat,	—	—	+30.048	
Correct height in fathoms,	—	—	684.157 × 6	
Height in feet by the barometer,		—	4104.942	
———— by the geom. method		—	4211.3	
Difference, or error of the barometer,			—106.4 = $\frac{252}{10000}$.	

Obſervation below at 1.

	Barom. N° 1. below at 1. In. Pts.	Therm. attached.	Therm. detached.	
	28.1310	62°.6	63°.6	
Correct for the diff. of the barometer,	— 42		57.0	heat at 3.
	28.1268		60.3	mean heat.
			39.7	ſtandard temperature.
			20.6	difference.

To collect theſe laſt experiments in one point of view.

	Feet.
The 1ſt ſeries gives for the error on every 1000 ft.	18.7
2d, — — —	16.9
3d, — — —	22.8
4th, — — —	23.5
5th, — — —	23.1
6th, — — —	25.2
The mean error, —	21.7

which agrees within two feet in a thouſand with the determination on Mont Saleve. This reſult then juſtifies my concluſion (in p. 44.) and proves that either the proportional gravity of air and quickſilver is now different from what it was, when M. DE LUC made his experiments, *viz.* from 1756 to 1760; or that his or my obſervations are defective. That my trigonometrical meaſurements were ſufficiently exact, *viz.* to within two or three feet, I think I have already ſhewn; and even that his were alſo. Within what limits my barometrical errors are to be found is not difficult to determine from what has been before premiſed. That the ſcale of Mr. DE LUC's barometer was leſs accurate than mine, is, I think, without a doubt; and indeed he never attempted a diviſion leſs than $\frac{1}{16}$th of a French line, or about $\frac{5}{1000}$

of an inch Engliſh: and yet when I conſider the number of his obſervations, and the unexampled diligence and care with which he made them, I am obliged to attribute the difference of our reſults to ſome other cauſe than that of inaccuracy. If then future experience ſhould demonſtrate, that the denſity of the atmoſphere with a given heat is invariable, or nearly ſo; while the preſſure of a whole column of it continues the ſame, we may perhaps ſearch for the cauſe of our diſagreement from hence, *viz.* the barometers of Mr. DE LUC were not ſufficiently near each other in an horizontal direction: mine were ſeparated from two to three miles; and his, I believe, at double or triple that diſtance. It may be ſuſpected, I am well aware, that the ſyphon conſtruction of Mr. DE LUC's barometer might occaſion this difference: let us ſee whether this be the caſe. Mr. DE SAUSSURE (whoſe inſtrument was of Mr. DE LUC's conſtruction, and made, as I underſtood, under his inſpection) obſerved at the top of the Mole, or at leaſt nearly on the ſame level with my barometer, as follows:

	Barometer In. L. 16ths.	Therm. attached. DE LUC's ſcale.	Therm. det. REAUM. ſcale.
	22 8 0	+1°+	+10°−
And in Engliſh meaſure and FAHRENHEIT's ſcale, — —	24.1570	56	54.2
Mr. DE SAUSSURE's barometer ordinarily ſtands higher than mine N° 2. by (q),	—.0117		
Correct for the diff. of our attached therm. 1°,	+ 26		
Mr. DE SAUSSURE's barometer corrected,	24.1479		
My barometer N° 2. ſee the firſt ſeries,	24.1437 —	57	54.8
Difference, — —	+ .0042 wholly inconſiderable.		

Our barometers may therefore be ſaid to have agreed exactly.

Mr. DE SAUSSURE made a ſecond compariſon juſt before we left the top of the mountain, which proved as follows.

	Barometer In. L. 16ths.	Therm. attached. DE LUC's ſcale.	Therm. detached.
	22 8 8	+4°	+11$\frac{2}{3}$°
Or reduced to Engliſh meaſure and ſcale,	24.2014	61.7	57.9
Mr. DE SAUSSURE's barometer ſtands higher than mine N° 2. —	—.0117		
Corr. for the diff. of our attached therm. 0°.7,	—.0018		
Mr. DE SAUSSURE's barometer corrected,	24.1879		
My barometer N° 2. ſee the ſixth ſeries,	24.1900	61.0	57
Difference, — —	—0.0021		

So that, in the firſt compariſon, his barometer at the top of the Mole ſtood higher than mine by +,004 inch; and in the laſt, lower by −,002; the mean is higher by

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(q) This we found by compariſons at the bottom of the mountain.

+,001

+,001, equal to about 10 inches in deducing the height of the mountain, a quantity wholly to be neglected. Finally, the mean of Mr. DE SAUSSURE's obſervations gives the defect of Mr. DE LUC's rules 21.9 in a thouſand. The conſtruction of the barometer had therefore no influence on this difference. But further, while Mr. DE SAUSSURE obſerved the height of the barometer on the Mole, Mr. DE LUC, the brother made a correſponding obſervation with a ſimilar inſtrument at Geneva. I ſhall relate this obſervation, computed after Mr. DE LUC's manner.

	In.	L.	16ths.	16ths of a line.	Log.	Heat of the air.	
						BEAUM. Therm.	DE LUC's Therm.
Mr. DE SAUSSURE, at 4 feet below the ſummit of the Mole, —	22	8	0				
Mr. DE SAUSSURE's barom. ſtands higher than Mr. DE LUC's ordinarily by,			$+1\frac{1}{2}$				
Thermometer attached +1°,			$-0\frac{3}{4}$				
Correct height on the Mole,	22	8	$0\frac{3}{4}$	$=4352.\frac{3}{4}$	6387587	+10	$-15\frac{2}{3}$
Mr. DE LUC, 78 feet above the lake, —	27	0	0				
Therm. attahced +6°, —			− 6				
	26	11	10	=5178	7141620	+15	− 4
Difference of the Log.	—	—			754.033	Sum	$-19\frac{2}{3}$
$19^{\circ}\frac{2}{3}\times\frac{750433}{1000}=$ the correction for the temperature,					−14.854		
Correct height in French toiſes,		—			739.179		
					× 6		
Height in French feet,	—	—			4435.074		
Mr. DE LUC's barometer above the lake of Geneva,					+78.		
Mr. DE SAUSSURE's barometer below the ſummit of the Mole, — — —					+ 4.		
And conſequently, the ſummit of the Mole above the lake, in French feet, — —					4517.		
Which reduced to Engliſh feet is, — —					4814.		
But, by a mean of my trigonometrical operations, this height is (*vide* chart) — —					4883.		
Difference, or error of the barometrical rules,					$-69.=\frac{14}{1000}.$		

This laſt obſervation ſerves at leaſt to ſhew, that the error I am contending for is on the defective ſide, though it gives the quantity of it ſomewhat leſs, but by no means deſerves that confidence which the other compariſons do; for, beſides that this ſingle obſervation may be concluded leſs deciſive, the trigonometrical meaſurement is alſo leſs accurate from the diſtance; and, laſtly, to ſuppoſe the ſtate of the atmoſphere preciſely the ſame with reſpect to weight in two places twenty miles aſunder, is, I am afraid, a *poſtulatum* too hazardous to grant. I therefore ſay, that all theſe obſervations confirm the ſame truth, that the atmoſphere is lighter than Mr. DE LUC preſumed it. What had already been done may ſeem ſufficient for the eſtabliſhment of this fact; for I have always held, that a few obſervations, well made and faithfully related, do more in the interpretation of nature, than a multitude of crude, careleſs, and immethodical experiments. But I have not done: I wiſhed to put this matter out of all doubt, and accordingly undertook another expedition to the ſummit of Mont Saleve, on the 18th of September, and in a colder temperature: the experiments then made, with their reſults, were as follows:

The difference of actual height by the two barometers was 2828.9 feet, the barometer N° 1. ſtanding higher than N° 2. by +,0038 inch, when compared at the bottom of the mountain.

Compariſon of the firſt ſeries.

Obſervation at the top of the mountain.			Obſervation at the bottom.		
Barom. N° 2. at the top.	Therm. attached.	Therm. detached.	Barom. N° 1. below.	Therm. attached.	Therm. detached.
In.	°	°	In.	°	°
25.6533	58.0	56.2	28.4040	58.1	58.8

Feet.

This gives for the height barometrically, 2755.6

But the true height was, — 2828.9

Difference, or error of the barometers, $-73.3=\frac{259}{10000}$.

Compariſon of the ſecond ſeries.

Obſervation at the top of the mountain.			Obſervation at the bottom.		
Barom. N° 2. at the top.	Therm. attached.	Therm. detached.	Barom. N° 1. below.	Therm. attached.	Therm. detached.
In.	°	°	In.	°	°
25.6550	56.2	57.0	28.4040	58.5	60.8

Feet.

This gives for the height barometrically, 2754.9

But the true height was, — 2828.9

Difference, or error of the barometers, $-74.0=\frac{262}{10000}$.

Compariſon of the third ſeries.

Obſervation at the top of the mountain.			Obſervation at the bottom.		
Barom. N° 2. at the top.	Therm. attached.	Therm. detached.	Barom. N° 1. below.	Therm. attached.	Therm. detached.
In.	°	°	In.	°	°
25.6620	56.2	57.2	28.4040	59.3	62.0

Feet.

This gives for the height barometrically, 2748.9

The height by the trigon. method was, 2828.9

Difference, or error of the barometers, $-80.0 = -\frac{282}{10000}$.

Compariſon of the fourth ſeries.

Obſervation at the top of the mountain.			Obſervation below.		
Barom. N° 2. at the top.	Therm. attached.	Therm. detached.	Barom. N° 1. below.	Therm. attached.	Therm. detached.
In.	°	°	In.	°	°
25.6600	56.4	57.4	28.4040	59.3	62.2

Feet.

This gives for the height barometrically, 2752.8

But the true height was, — 2828.9

Difference, or error of the barometers, $-76.1 = \frac{269}{10000}$.

In theſe compariſons I have not inſerted the whole of the computation, as that may eaſily be made by any perſon at leiſure. Finally, the mean of theſe four laſt ſeries gives for the error on 1000 feet, 26.8. I think I have now ſhewn, that the error actually exiſts; it remains that we determine preciſely the quantity of it. For this purpoſe it will be proper to collect all the preceding obſervations in one point of view.

Table of the reſult of all the barometrical experiments.

Place of obſervation.		True height trigonometrically.	Height by the barometers.	Mean heat.	Error in feet.	Error in 1000 feet.	Mean error in 1000 feet.
Mont Saleve,	1	2831.3	2775.2	69.4	— 56.1	—19.8	—23.1
	2	——	2763.2	68.5	— 68.1	—24.0	
	3	——	2759.4	67.2	— 71.9	—25.4	
At the Mole,	1	4211.3	4132.7	58.3	— 78.6	—18.6	—21.7
	2	——	4140.1	58.9	— 71.2	—16.9	
	3	——	4115.1	59.5	— 96.2	—22.8	
	4	——	4111.9	60.0	— 99.4	—23.5	
	5	——	4113.7	60.5	— 97.6	—23.1	
	6	——	4104.9	60.3	106.1	—25.2	
Mont Saleve,	1	2828.9	2755.9	57.5	— 73.3	—25.9	—26.8
	2	——	2754.9	58.9	— 74.0	—26.2	
	3	——	2748.9	59.6	— 80.0	—28.2	
	4	——	2752.8	59.8	— 76.1	—26.9	

Mean of all, 23.6, and the temperature 61°.4.

Place of obſervation.		True height trigonometrically.	Height by the barometers.	Mean heat.	Error in feet.	Error in 1000 feet.	Mean error in 1000 feet.
The Mole, from two obſervations of Mr. DE SAUSSURE, —		4211.3	——	—	— 92.	—21.8	—16.2
The ſame by Mr. DE SAUSSURE, and Mr. DE LUC, at Geneva,		4883.	4814.	—	— 69.	—14.	
According to Mr. DE LUC's own obſervations, ſee *Recherches ſur l'Atmoſphere,*	the Mole,	4882.8	4860.	—	— 22.8	— 4 7	
	the Dole,	4292.7	4210.	—	— 82.7	—19.5	
	the Buet,	8893.6	8770.	—	—123.7	—13 9	
	Mt Blanc,	14432.5	14093.	—	—339.5	—23.5	

The titles of the columns are ſufficiently clear to make a farther explanation of this table unneceſſary; and it appears, I think inconteſtably, upon taking a mean of my thirteen obſervations (and I ſhall here conſider only my own) on Mont Saleve and the Mole, that this error is about $23\frac{1}{2}$ feet on every thouſand; that is, the rules of Mr. DE LUC give the height by ſo much too little. At the bottom of the foregoing table I have ſubjoined ſix other compariſons, ſome of them from Mr. DE LUC's own obſervations, as recorded in his valuable work; which however I muſt add, are certainly of leſs authority in this inquiry, as they were made with barometers a great way diſtant from each other, *viz.* near thirty miles: beſides which, the geometrical heights are, for the ſame reaſon, not ſo accurately aſcertained. I have, however, ventured to make what uſe I could of them, *viz.* to ſhew that theſe two give a reſult on the ſame ſide, though not exactly the ſame; and to urge the neceſſity of a certain vicinity in thoſe obſervations from whence a theory is to be deduced.

Shall I be permitted to adduce another proof, in confirmation of what has been advanced? When I firſt took up the conſideration of meaſuring altitudes in the atmoſphere with the barometer, and had heard only of Mr. DE LUC's labours, it occurred to me, that there was a much more ſimple method of arriving at this theory, than either he or I have ſince purſued. It was this; to determine hydroſtatically the ſpecific gravities of air(r) and quickſilver, with a given temperature and preſſure; the increaſe of volume, or change of gravity, with a given increaſe of heat being ſuppoſed to be known by the experiments of **BOERHAAVE**(s) and **HAWKESBEE**(t), which might be farther examined by ſimilar ones; and preſuming that the geometrical ratio in the air's denſity, as you advance upwards from the earth's ſurface, had been ſufficiently demonſtrated(u). For the proportional gravity of quickſilver to air will expreſs inverſely the length of two equiponderant columns of theſe fluids, that is, when the columns are taken infinitely ſmall(x). With theſe ideas I made the following experiment. I cauſed a glaſs veſſel to be blown ſomething like a Florence flaſk, or rather larger; to the neck of this was adapted a braſs cap with a valve opening outwards, and made to ſcrew on or off, together with a male ſcrew, by which it was fixed to an excellent pump of Mr. NAIRNE'S conſtruction, and exhauſted of its air, or at leaſt rarified to a known degree: the veſſel was then carefully weighed with a ſenſible balance, and again after the air was re-admitted; the difference gave the weight of the air that had been exhauſted. After having repeated this two or three times, the veſſel was exactly filled with water as far as the valve, which had been the term of capacity for the air; this was done by ſcrewing on the cap till the ſuperfluous water oozed all out, and upon inverting the veſſel there appeared not the leaſt ſign or bubble of air; I therefore concluded, that the volume of water was preciſely the ſame as had been the volume of air, a circumſtance that ſhould be accurately attended to. It was then carefully weighed, and compared with its weight when full and deprived of its air. It will readily be ſeen, that I had then the ſpecific gravity of the two fluids, upon ſuppoſition that the figure of the glaſs had not altered by

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(r) It may ſeem particular that I ſhould propoſe an experiment ſuppoſed to be very well known, and which hardly any elementary treatiſe on chemiſtry or experimental philoſophy will not furniſh us with an example of; the weight of a given quantity of air. **BOYLE**, **HALLEY**, **HAWKESBEE**, **HALES**, each of them have tried it, and many others ſince their time: but the misfortune is, all theſe experiments have been but groſs approximations, without due attention to the heat; and yet the determination of **HAWKESBEE** ſeems to have been followed by one-half of Europe in Pneumatical reſearches. Indeed I only know of one experiment that has the leaſt title to preciſion, and that is Mr. **CAVENDISH**'s, briefly related in the LVIth volume of the Philoſophical Tranſactions.

(s) Elementa Chemiæ.

(t) Phyſico-mechanical Experiments.

(u) **COTES**'s Hydroſtat. Lectures, *et alibi*.

(x) I am not ſorry to anticipate the reader's remark here, that this obſervation is not new; ſince I find that I have been treading the ſame ſteps with Mr.

Mr. BOYLE and Dr. HALLEY, who both made uſe of this method; the one with a view to determine the limits of the atmoſphere; and the other the height of Snowden.

by preſſure during the experiment; and this effect may be preſumed to have been the moſt ſenſible, when the veſſel was filled with water, the preſſure at that time being from within. To aſſure myſelf of this, I let in a ſmall quantity of air, which formed a bubble of about one-third of an inch in diameter, and upon immerging the glaſs in another veſſel of water, whereby the preſſure within was counterpoiſed by a preſſure without, the bubble ſeemed to contract itſelf by a quantity, as I found afterwards, equal to about two grains in weight, or $\frac{1}{8000}$ of the whole contents. I therefore concluded, that this correction was hardly worth taking notice of, and ſtill leſs the effect from external preſſure when the glaſs was exhauſted. At every operation the height of the barometer and thermometer (placed cloſe to the veſſel when the air was weighed) was noted down, together with the height of the pump-gage, which, compared with the barometer in the room, ſhewed the quantity exhauſted. The reſult of the experiment was as follows, the barometer in the room ſtanding at 29.27 inches, and the heat of the room 53°.

	~~Feet.~~ Grains
The bottle empty or exhauſted till the gage ſtood at 29.15 inches weighed (determined from four different trials, and the balance turning with $\frac{1}{15}$ of a grain) — — —	2657.40
Increaſe of weight when filled with air, from four trials certain to $\frac{1}{10}$ of a grain — — —	+ 16.13
Bottle filled with water, whoſe heat was 51°, — —	16220.00
Weight of the water, excluſive of the bottle, — —	13562.60

But the bottle was exhauſted only in the proportion of 29.15 inches to 29,27 inches; therefore if a perfect *vacuum* could have been made, the difference of weight would have been 16.22 grains inſtead of 16.13 grains. Again, the water was colder than the air by 2°; the one being 53°, and the other only 51°: now water, from former experiments, I find to expand about $\frac{1}{10000}$ with 2° of heat; therefore, if the water had been of the ſame temperature with the air that was examined, the weight of an equal volume would have been only 13558,5 grains; and laſtly, 13558.5 divided by 16.22 gives 836(y), and by ſo much is water heavier than air in theſe circumſtances.

(y) HAWKESBEE's experiments made the air 850 lighter than water, the barometer being at 29 7; and Dr. HALLEY ſuppoſed it about 800. Mr. CAVENDISH, in weighing 50 grains of air, when the barometer was at 29¼, and the thermometer at 50°, concluded the ſpecific gravity of air to be about 800 alſo. Now my experiment, reduced to the ſame circumſtances with his, would give 817 for this gravity, no great difference in an affair of ſuch delicacy.

By

By former experiments I find the ſpecific gravity of the quickſilver of my barometers, compared with rain-water in 68° of heat, as, — — — —	13.606 to 1
And 68°—53°=15°, correct therefore for 15° of expanſion of quickſilver, — — —	+.018
Correct for 15° of expanſion of air; — —	—.031
True ſpecific gravity of quickſilver, with 53° of heat,	13.594
Which multiplied by the ſpecific gravity of air, —	×836
Gives for the comparative gravity of quickſilver and air, when the barometer is at 29.27, and the thermometer 53°, —	11364.6
	Feet.
And laſtly, $\frac{1}{10}$th of an inch of quickſilver, when the barometer ſtands at 29.27 inches (*viz.* from 29.22 inches to 29.32 inches) with the temperature 53°, is equal to a column of the atmoſphere of, —	94.7
This quantity, according to my barometrical obſervations, is, —	93.83
———————— to Mr. DE LUC's rules, — —	91.66

We ſee here then that the ſtatical experiment agrees with the reſult of my barometrical ones to within about 11 inches in 100 feet, and I am not ſure that it is not ſtill capable of much farther preciſion; and though perhaps alone it might carry with it, to ſome perſons, a leſs concluſive teſtimony, who reluctantly reaſon from the little to the great, yet, in conjunction with what has been before ſhewn, I think it has conſiderable weight; and I am the leſs inclined to reject ſuch an indirect method of proof, as I have the great authorities of HALLEY and NEWTON on my ſide(x).

I have

(x) "Ce qu'il y a d'eſſentiel à obſerver ici," ſays Mr. DE LUC, "et vraiment digne de remarque, c'eſt que par la ſeule connoiſſance des peſanteurs ſpecifiques de l'air et du mercure, HALLEY eſt parvenu à une regle trés

" approchante

I have thus endeavoured to ſhew then that the error of the theory is $= \frac{236}{10000}$ when the temperature of the air is 61°.4 (ſee the table of the reſult of the obſervations). It remains therefore, finally, that we deduce a rule, the error of which ſhall be nothing, *viz.* to find the temperature wherein the difference of the logarithms of the heights of the barometer, taken to four places of figures, will give the true difference of elevation in Engliſh fathoms. Previous to this inveſtigation, with which I intend to conclude this paper, it will be neceſſary to remark, that by repeated experiments with the barometer, I find a ſmall difference in the equation for the expanſion of air by a change of temperature, and even in that of quickſilver from the ſame cauſe, from what Mr. DE LUC's obſervations have given it (a). I ſhall

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" approchante de celle, qu'un grand nombre d'obſervations du barometre dans " les Cordelières ont dicté depuis à M. BOUGUER: cependant malgré l'appui " que ces experiences ſe prêtent reciproquement, on verra qu'elles etoient encore " bien eloignées de fournir une regle generale." Recherches ſur l'Atmoſphere, ſect. 267.

(a) He indeed made his experiments on the atmoſphere itſelf with the barometer, in order to determine the variations of its denſity; but ſince it appears that the abſolute denſity of this fluid is different from what he ſuppoſed it, it is no bold conjecture to preſume that the degree of its variation ſhould be different alſo; and to aſcertain this point, I have preferred the inſtrument above-mentioned to the method uſed by Mr. DE LUC, how direct ſoever his may ſeem; for in determining minute quantities or equations, we muſt not embarraſs ourſelves with the compound effect of too many cauſes at a time.

not here trouble the reader with the experiments at large, too ſimple in themſelves to deſerve ſuch a detail, unleſs a future occaſion ſhould render that neceſſary, as the methods here uſed may be met with amongſt HAWKESBEE's or Mr. BOYLE's experiments; and content myſelf with relating only the reſult of the different trials.

1000 parts of air of the temperature of freezing and preſſure of 30½ inches, increaſed in volume by an addition of 1 degree of heat on FAHRENHEIT's thermometer as follows:

Obſervations.		Number of degrees the air was heated.	Expanſion for 1° in 1000ths of the whole.	
With the firſt manometer,	1	14.6°	2.30	Mean from the firſt manometer 2.44.
	2	32.2	2.43	
	3	40.3	2.48	
	4	46.6	2.45	
	5	49.7	2.48	
	6	51.1	2.51	
	7	23.7	2.36	
	8	13.1	2.24	
With another manometer,	9	22.0	2.38	Mean from the ſecond manometer 2.42
	10	28.0	2.50	
	11	21.5	2.34	
	12	30.1	2.44	
	13	22.6	2.44	

The mean of theſe two ſorts of obſervations, made with different inſtruments, is 2.43, *viz.* 1000 parts of the air at freezing become by expanſion from 1° of heat equal $\overset{\text{Pts.}}{1002.43}$ or $\overset{\text{Pts.}}{1002.385}$ with the ſtandard temperature 39°.7. Mr. DE LUC's experiments reduced give this quantity equal $\overset{\text{Pts.}}{1002.23}$(b) (ſee Tranſ.). It may be imagined, that I ſhould have had a more accurate concluſion by making theſe obſervations in greater differences of temperature than what is ſhewn in the ſecond column of the above table; but it did not appear ſo to me. On the other hand, I found that it was abſolutely neceſſary that the ſame heat ſhould be kept up for ſome hours together, in order that I might be ſure that the air within the inſtrument, the glaſs tube that contained it, and the air without it, all had acquired the ſame uniform temperature, which in my room I found not very eaſy to effect in heats greater than 70° or 80°. I have therefore preferred repeating the experiment with ſmall differences of heat; but ſuch, however, as will include almoſt all the temperatures in which barometrical obſervations are likely to be made, *viz.* from 32° to 83°.

(b) It has generally been ſuppoſed, that air expands $\frac{1}{480}$ with each degree of the thermometer, commencing from the mean temperature 55°; and, in conſequence of this, aſtronomers have computed tables for correcting their mean refractions; but, upon reducing the reſult of my obſervations to the temperature 55°, we ſhall have the correction of the refraction for 1° $= \frac{243}{105589}$ or $\frac{1}{435}$. Now according to Mr. DE LUC this equation is $\frac{223}{103345} = \frac{1}{463}$, which would produce a difference of about 4″ in the corrected refraction, upon an altitude of 5°, with the temperature 35°. If my numbers may be ſuppoſed to deſerve equal confidence, the error of the tables in common uſe, in the above circumſtances, would amount to only half that quantity, and therefore probably will be thought ſcarce worth correcting. I have mentioned this in order to obviate the concluſions that have been drawn by ſome perſons from Mr. DE LUC's theory.

It has been ſuſpected, in conſequence of ſome experiments made by a very ingenious member of this Society, that air does not expand uniformly with quickſilver; or that the degrees of heat ſhewn by a quickſilver-thermometer would be expreſſed on a manometer, or air-thermometer, by unequal ſpaces in a certain geometrical ratio. I do not deny this propoſition; but I have alſo very little reaſon to aſſent to it, if I may truſt my own experiments, which certainly evince that this ratio, if not truly arithmetical, is ſo nearly ſo as to occaſion no ſenſible error in the meaſuring of heights with the barometer; and that is all I contend for. The ſmall differences that are ſeen in the above table of this expanſion, deduced from a mean of 14° or of 40°, I would attribute rather to the errors of obſervation than to any actual irregularity in nature. If, however, this progreſſion be inſiſted upon, it ſhould ſeem, that the degree of the air's expanſion increaſes with an increaſe of heat; and that the difference of volume or denſity from 1° of heat, any where within the limits above-mentioned, would be about one part in five thouſand from what I take it at a mean. I ſhould not have inſiſted ſo long on this circumſtance, but in reſpect to the known accuracy of the author of this hypotheſis. Neither do I find any reaſon to believe, that the expanſion of air varies with its denſity. I have tried air whoſe denſity or preſſure was equal to 23¾ inches, and alſo to forty inches; but the dilatation, with equal volumes and equal degrees of heat, was very nearly the ſame in both caſes. I might add a great deal more on theſe manometrical experiments, but I am afraid it would be more tedious than uſeful. I proceed therefore to the expanſion of quickſilver.

This experiment was made with a tube, ſomething like a thermometer, but conſiderably larger than the ordinary ſize, and open at one end; it was filled with quickſilver to a certain height, and then expoſed to the temperatures of freezing and boiling repeatedly, the barometer being at 30 inches: the difference of the volume in each inſtance was determined afterwards by accurately weighing the contents. I thus found, that if the quickſilver at freezing be ſuppoſed to be divided into 13119 parts, the increaſe of volume by a heat of boiling water became equal to 208 of theſe parts $= \frac{10}{637}$, and $\frac{10}{637} \times \frac{1}{180} = \frac{1}{11466}$; and ſuch would be the expanſion for each degree of the thermometer, commencing from the freezing point, $=0{,}00262$ inch on a column of 30 inches of the barometer, if the glaſs had ſuffered no expanſion during the experiment. This, however, has been found to be with 180° of heat $= \frac{1}{400}$ in ſolidity (*viz.* the cube of its longitudinal expanſion) and $\frac{1}{400} \times \frac{1}{180} = \frac{1}{72000} = 0{,}00042$ inch, for the effect of the expanſion of the glaſs for 1° upon a column of 30 inches; this added to the quantity before found, which was only the exceſs of the greater expanſion above the leſs, gives for the true equation for each degree 0,00304 inch when the barometer ſtands at 30 inches (c). Mr. DE LUC'S correction in this caſe was 0,00312; a difference ſo ſmall that I ſhall take no notice of it as to its influence upon our obſervations. It may deſerve a remark here, that this equation rigorouſly taken is variable according to the height of the thermometer; for 1°, which at freezing is $= \frac{1}{9891}$ of the whole volume, at the temperature 82° becomes $\frac{1}{9941}$, a difference indeed that may fairly be neglected, and which I neglect myſelf; yet I cannot help obſerving, in juſtice to Mr. DE LUC, that his method of reducing his barometers always to the ſame ſtandard temperature, was free from the error I am ſpeaking of.

(c) It has been ſuſpected, and I believe will appear from very good obſervations, which however I never made myſelf, that the expanſion of quickſilver in the barometer is not directly as the heat ſhewn by the thermometer, but in a ratio ſomething different, owing to ſome of the quickſilver being converted into an elaſtic vapour in the *vacuum* that takes place at the top of the Torricellian tube, which preſſes upon the column of quickſilver, and thus counteracts in a ſmall degree the expanſion from heat. It does not, however, appear to be a conſiderable quantity, not amounting to above one-ſixteenth of the whole expanſion in a range of 40° of temperature; I ſhall therefore venture to conſider this equation as truly uniform, ſince the error on ten thouſand feet would not amount to five.

To conclude, the defect of Mr. DE LUC's rules being ſuppoſed $\frac{236}{10000}$, or, which comes to the ſame thing, the correction being $+\frac{2417}{100000}$, when the temperature of the air is 61°.4, and the true expanſion of the air for each degree being $\frac{239}{100000}$ when the heat is 39°.7; required to find the temperature wherein the difference of the logarithms ſhall give the true height in Engliſh fathoms, that temperature, according to Mr. DE LUC, being 39°.74, and the expanſion $\frac{223}{100000}$.

Let T be the temperature 61°.4; S Mr. DE LUC's ſtandard temperature; E the expanſion for 1°; e the ſame, according to Mr. DE LUC; α the ſuppoſed correction of the rules, and x the temperature ſought. We have then the following formula, $\frac{\overline{T-S} \times \overline{E-e}^{(d)} - \alpha}{E} = S - x$, wherein proceeding with the above numbers $S-x$ comes out

8°.50

(d) This ſign is negative, becauſe the aſſumed expanſion e is leſs than the true one E, and conſequently tended to increaſe the apparent error of the rules; had it been greater, α would have been +.

$8^\circ.50$, and conſequently $x=31^\circ.24$ the temperature required; which, if it ſhould be thought convenient, may be conſidered as the freezing point.

In the whole of the above inquiry I have taken no notice of the effect of gravity upon the particles of the air at different diſtances from the earth's center, which ſhould doubtleſs enter into the account, and which would occaſion the denſity of the atmoſphere to decreaſe in a ratio ſomething greater than the preſent theory admits of. In a height of four Engliſh miles Dr. HORSLEY finds (Phil. Tranſ. vol. LXIV.) that the diminution of denſity or volume from the accelerative force of gravity would be only $\frac{1}{500}$ part of the whole, or about 48 feet; and I may add to this, that this effect will be in the duplicate ratio of the heights, ſo that at one mile high it becomes only three feet. A like effect takes place alſo below the ſurface of the earth, as in meaſuring the depths of mines, &c. with this difference, that here it is but half the quantity; in the former inſtance gravity within the earth being ſimply as the diſtance from the center; they are both of them, however, circumſtances that deſerve no attention in practice.

This would be the place for me to enumerate the many deſiderata, beſides thoſe already hinted at, that ſtill remain for the perfection of this theory; ſuch as the laws of heat, that obtain in the different regions of the atmoſphere; the effects of moiſture, winds, the electric fluid, together with the weight and qualities of the air in different countries, &c.; that at the ſame time that I am congratulating the preſent age on one of the moſt brilliant diſcoveries in natural philoſophy, I may be underſtood alſo to encourage every lover of ſcience to ſtill farther enquiries in a branch of knowledge no leſs uſeful than ingenious; particularly in a kingdom wherein, from its commercial intereſts, and in conſequence its many inland navigations, every improvement in hydroſtatics, the art of levelling, or geometry, cannot but tend conſiderably to the public benefit. The ſources of ſcience are not eaſily exhauſted; multitudes of them remain wholly unexplored. If novelty can afford a charm, the path I am ſpeaking of, till of late, has been the leaſt frequented; witneſs the freſh, important truths in Pneumatical reſearches that, from zeal and faſhion, every day's experience affords. I might here offer too a tribute of applauſe (and am ſure in concert with this whole aſſembly) juſtly due to the indefatigable labours of him whoſe ſteps I have purſued; but I am convinced he will rather hear me acknowledge our obligations to the ancients than any panegyric of himſelf. Be the benefit we receive from them our encouragement to proceed.

Multum egerunt, qui ante nos fuerunt, ſed non peregerunt: multum adhuc reſtat operis, multumque reſtabit; nec ulli nato poſt mille ſæcula præcludetur occaſio aliquid adhuc adjiciendi." SEN. Epiſt. 64.

PART II.

IN the ſubſequent pages, which I have now the honour of laying before the Royal Society, I have drawn up, and I hope in a form the moſt commodious, the neceſſary tables and precepts for calculating any acceſſible heights or depths from barometrical obſervations, and without which I thought the preceding memoir would be incomplete; referring, however, to that for the proofs or elements from whence the tables have been computed. And herein I have endeavoured to adapt myſelf to the capacity of ſuch perſons as are but little converſant with mathematical computations, but who may have frequent opportunities of contributing ſomething to the advancement of ſcience by experiments with this uſeful inſtrument, which is now become nearly in as common poſſeſſion as a pocket watch. I have induſtriouſly avoided the method of logarithms, propoſed by Dr. HALLEY, and adopted by Mr. DE LUC, both becauſe ſuch tables are not in the hands of every body, and becauſe I have perceived that many perſons of a philoſophical turn, though ſkilled only in common arithmetic, have been frightened by the very name: a method leſs popular, however elegant, would have been leſs generally uſeful. To theſe tables is ſubjoined a liſt of ſeveral altitudes, as determined by the barometer: this will ſerve to ſhew the uſe I have made of the inſtrument, and will at the ſame time exhibit the level of a great number of places in France, Savoy, and Italy, and, as I think, be no improper ſupplement to exemplify the rules. It might have been expected that I ſhould have ſaid ſomething on the theory of barometrical obſervations, and have laid down the laws and principles on which it depends; but as that has been ſo amply done by other writers of inconteſted authority, I ſhall content myſelf with inſerting only the following propoſitions.

1ſt, The difference of elevation of two places may be determined by the weight of the vertical column of the atmoſphere intercepted between them.

2d, If then the weight of the whole atmoſphere at each place can be aſcertained, the weight of this column, *viz.* their difference, will be known.

3d, But the height of the quickſilver in the barometer expreſſes the total weight of the atmoſphere in the place of obſervation; the difference, therefore, of the height of the barometer, obſerved in two places at the ſame time, will expreſs the difference of elevation of the two places.

4th, But further, the weight of this column of the atmoſphere is liable to ſome variations, being diminiſhed by heat, and augmented by cold; and again, a ſimilar alteration takes place in the column of quickſilver, which is the meaſure of this weight.

5th, If then the degree of theſe variations can be determined, and the temperature of the air and quickſilver at the time of obſervation be known, the weight of this column of air, or the difference of elevation of the two places, will be concluded as certainly as if the gravity of theſe two fluids, with all heats, remained invariably the ſame: this is the whole myſtery of barometrical meaſurement.

APPLICATION.

The height of the barometer in Engliſh inches at any two places at the ſame inſtant, and the heat (according to FAHRENHEIT's thermometer) to which it is expoſed, being known, together with the temperature of the air at each place, obſerved with a ſimilar inſtrument; required the difference of elevation of the two places in Engliſh feet.

RULE.

Precept the 1ſt, With the difference of the two thermometers that give the heat of the barometer (and which, for diſtinction ſake, I ſhall call the attached thermometers(e)) enter table I. with the degrees of heat in the column on the left hand, and with the height of the barometer in inches, in the horizontal line at the top; in the common point of meeting of the two lines will be found the correction for the expanſion of the quickſilver by heat, expreſſed in thouſandth parts of an Engliſh inch; which added to the coldeſt barometer, or ſubtracted from the hotteſt, will give the height of the two barometers, ſuch as would have obtained had both inſtruments been expoſed to the ſame temperature.

(e) It is ſcarce neceſſary to remark, that, in order to make good concluſive obſervations, it is proper to be furniſhed with two barometers, and four thermometers; *viz.* one attached or inſerted in the frame of each barometer; and the other two detached from them, in order to take the heat of the open air; for it will ſeldom be found, that the thermometer in the frame of the barometer and that in the air will ſtand at the ſame point, and for a very evident reaſon.

Precept the 2d, With theſe corrected heights of the barometers enter table II. and take out reſpectively the numbers correſponding to the neareſt tenth of an inch; and if the barometers, corrected as in the firſt precept, are found to ſtand at an even tenth, without any further fraction, the difference of theſe two tabular numbers (found by ſubtracting the leſs from the greater) will give the approximate height in Engliſh feet. But if, as will commonly happen, the correct height of the barometers ſhould not be at an even tenth, write out the difference for one entire tenth, found in the column adjoining, intitled *Differences*; and with this number enter table III. of proportional parts in the firſt vertical column to the left hand, or in the 11th column, and with the next decimal following the tenths of an inch in the height of the barometer (*viz.* the hundredths) enter the horizontal line at the top, the point of meeting will give a certain number of feet, which write down by itſelf; do the ſame by the next decimal figure in the height of the barometer (*viz.* the thouſandths of an inch) with this difference, ſtriking off the laſt cypher to the right hand for a fraction; add together the two numbers thus found in the table of proportionable parts, and their ſum ſubduct from the tabular numbers juſt found in table II.; the differences of the tabular numbers, ſo diminiſhed, will give the approximate height in Engliſh feet.

Precept the 3d, Add together the degrees of the two detached or air-thermometers, and divide their ſum by 2, the quotient will be an intermediate heat, and muſt be taken for the mean temperature of the vertical column of air intercepted between the two places of obſervation: if this temperature ſhould be 31°$\frac{1}{4}$ on the thermometer, then will the approximate height, before found, be the true height; but if not, take its difference from 31°$\frac{1}{4}$, and with this difference ſeek the correction in table IV. for the expanſion of air, with the number of degrees in the vertical column on the left hand, and the approximate height to the neareſt thouſand feet in the horizontal line at the top; for the hundred feet ſtrike off one cypher to the right hand; for the tens ſtrike off two; for the units three: the ſum of theſe ſeveral numbers added to the approximate height, if the temperature be greater than 31°$\frac{1}{4}$, ſubtracted if leſs, will give the correct height in Engliſh feet. An example or two will make this quite plain.

EXAMPLE I.

Let the height of the barometer, obſerved at the bottom of a mountain be 29.4 inches, the attached thermometer 50°, and the heat of the air 45°; at the ſame time that at the top of the mountain the barometer is found to ſtand at 25.190 inches, the attached thermometer at 46°, and the air-thermometer at 39°½; required the height of the mountain in Engliſh feet. Set the numbers down in the following order:

Obſer-

Obſervation at the bottom.

	Barometer.	Therm. attached.	Therm. in the air.
	In.		
	29.400	50°	45°
		46	
Diff. of the two attached thermometers,		4	

Obſervation at the top.

	Barom.	Therm. attached.	Therm. in the air.
	In.		
	25.190	46°	39°½
Correct for the diff. of the two attached therm. *viz.* 4°,	+ 10		45
			2)84½(42¼ mean heat.
			31¼ ſtandard heat.
Height of the uppermoſt barometer, reduced to the ſame heat as the lowermoſt, *viz.* 50°, — —	25.200		11 difference.

Correct for 11°, ſee tab. IV.	
on 4000 feet	106.9
on 16 —	+ 5
or on 4016	+ 107.4

	In.	Feet.
Tabular number, ſee tab. II. correſponding to, —	25.200	= 6225.0
The ſame, correſponding to	29.400	= 2208.2
Approximate height in feet,		4016.8
Correction for 11° of heat on 4016 feet, add, — —		107.4
Correct height of the mountain — — —		4124.2

Now the difference of the attached thermometer 50° and 46° is = 4°; and againſt this number, in table I. in the column for 25 inches (being the height of the barometer in this caſe) I find 10, which added to 25.190, as this barometer was the coldeſt, gives 25.200 inches for the height of the uppermoſt barometer reduced to the ſame heat as the lowermoſt: and in table II. oppoſite to 25.200 inches and 29.400 inches, I find reſpectively 6225.0 and 2208.2; their difference 4016.8 is the approximate height in feet. The degrees on the thermometer in the open air, 39°½ and 45° being then added together, and afterwards divided by 2, give for the mean temperature of theſe obſervations 42°¼, or 11° above the ſtandard temperature, 31°¼: and laſtly, the correction for 11°, in table IV. on 4000 feet I find = 106.9, and on 16 feet = 0.5; that is, 107.4 feet equal the whole correction, which added to 4016.8 gives 4124.2 feet for the correct height of the mountain.

EXAMPLE II.

Suppoſe the height of the barometer at the top of a rock had been obſerved at 24.178, the attached thermometer at 57°.2, the air-thermometer at 56°; the barometer below at 28.1318 inches, the attached thermometer 61°.8, the detached one 63°.9; what is the height of the rock?

Obſervation at the bottom.

	Barometer.	Therm. attached.	Therm. detached.
	In.		
	28.1318	61°.8	63°.9
		57.2	
Difference of the two attached thermometers,		4.6	

Obſervation at the top.

	Barom.	Therm. attached.	Therm. detached.
	In.		
	24.1780	57°.2	56°.0
Correct for the diff. of the two attached therm. viz. 4°6,	+0112		63.9
			2)119.9(59.95 mean heat.
Height of the uppermoſt barom. reduced to the ſame heat as the lowermoſt, namely 61°.8,	24.1892		31.24 ſtandard temp.
			28.71 difference.

			Feet.	Diff.
Tabular number, correſponding to,	24.1000		7388.0	107.9
The ſame, ſee tab. III.	800	86.0	−95.9	
	90	9.7		
	2	.2		
	24.1892		7292.1	
Tabular number, correſponding to,	28.1000		3386.6	92.6
The ſame, ſee tab. III.	300	28.0	−29.6	
	10	0.9		
	8	0.7		
	28.1318		3357.0	
And 3357.0 feet taken from	—		7292.1	
Leaves the approximate height in feet,			3935.1	
Correction for 28°7 of heat on 3935 ft.			+274.3	
Correct height of this mountain,			4209.4	

Correct for 28°.7, ſee tab. IV.		
28° on	3000 =	204.1
	900 =	61.2
	35 =	2.4
0.7 on	3000 =	5.1
	900 =	1.5
	35 =	0.0
28.7 on	3935	274.3

This laſt obſervation was actually made, and the height geometrically was determined to be 4211.3 feet, not quite two feet different. In this example it will be obſerved, that as the height of the barometer is ſet down to four places of decimals; the tabular numbers, anſwering to every tenth only, are corrected by means of table III. of proportional parts, for the remaining decimals 8, 9, and 2, in one place; and 3, 1, 8, in the other; and their ſum is ſubducted from the numbers found in table II. And laſtly, that in finding the correction for 28°.7 of heat, the fraction $\frac{7}{10}$ is conſidered as ſo many units, and another decimal is ſtruck off; thus the correction on 3000 feet for 7° is 51; but for $\frac{7}{10}$ it becomes 5.1, and ſo of the reſt.

EXAMPLE III.

In the upper gallery of the dome of St. Peter's church at Rome, and 50 feet below the top of the croſs, I obſerved the barometer, from a mean of ſeveral obſervations, 29.5218; the thermometer attached being at 56°.6, and the detached one at 57°; at the ſame time that another, placed on the banks of the Tyber one foot above the ſurface of the water, ſtood at 30.0168, the attached thermometer at 60°.6, and the detached one at 60°.2; what was the total height of this building above the level of the river?

Obſer-

Obſervation below, at one foot above the Tyber.

	Barometer.	Therm. attached.	Therm. detached.
	In.		
	30.0168	60°.6	60°.2
		56.6	
Difference of the two attached thermometers,		4.0	

Obſervation above, in the gallery of St. Peter's church.

	29.5218	56.6	57.0	
Correct for the diff. of the two attached therm.	+ 120		60.2	
			2)117.2(58.60	mean heat.
Height of the uppermoſt barom. reduced to the heat of the lowermoſt *viz.* 60.5, —	29.5338		31.24	ſtandard temp.
			27.36	difference.

			Feet.	Diff.
Tabular numbers correſponding to, —	29.5000		2119.7	88.2
	300	26.4	—29.7	
	30	2.6		
	8	7		
	29.5338		2090.0	
Tabular numbers correſponding to, —	30.0000		1681.7	86.7
	100	8.7	—14.6	
	60	5.2		
	8	7		
	30.0168		1667.1	
			2090.0	
Approximate height,			422.9	
Correction for 27°4 of heat on 422 feet,			+28.0	
Difference of height of the barometers,			450.9	
Loweſt barom. ſtood 1 foot above the river,			+1.0	
Top of the croſs above the gallery was,			+50.0	
Total height of the top of the croſs above the river Tyber, —			501.9	
The ſame meaſured the ſame day geometrically was, — —			502.2	

Correction for 27°.4		
27° on	400 =	26.2
	22 =	1.4
0.4 on	400 =	.4
27.4 on	422 =	28.0

When the difference of the heights of the quickſilver in the two barometers happens not to exceed $\frac{1}{10}$ or even $\frac{2}{10}$ of an inch (and this will frequently be the caſe in levelling flat countries, or meaſuring ſmall heights) in ſuch circumſtances the moſt convenient way of reducing the obſervations will be by means of the column of differences only; thoſe numbers expreſſing the length of a column of the atmoſphere which correſponds to $\frac{1}{10}$ of an inch of quickſilver, at any aſſigned height of the barometer.

EXAMPLE IV.

Suppoſe the following obſervations had been made at the top and bottom of any eminence; *viz.* at the top, barometer 29.985 inches, attached thermometer 70°5, detached thermometer 76°; and below, barometer at 30.082, attached thermometer 71°, and the detached one 68°; what was the height of the eminence?

Obſervation below.

	Barometer.	Therm. attached.	Therm. detached.
	In.	°	°
	30.0820	71.0	68.0
		70.5	
Difference of the two attached therm.		0.5	

Obſervation at the top.

	Barometer.	Therm. attached.	Therm. detached.
	In.	°	°
	29.9850	70.5	76.0
Correct for 0°.5 of heat,	+.0015		68.0
Take — —	29.9865		2)144.0(72.0 mean heat.
From — —	30.0820		31.2 ſtandard temp.
Remains the difference or fall of quickſilver in the barometer, —	0.0955		+40.8 difference.

The difference for $\frac{1}{10}$ at 30 inches = 86.7 feet.

			Feet.
Therefore, for 0900	—	—	78.0
0050	—	—	4.3
0005	—	—	0.4

Correction for 41°.

	Feet.	Ft.
41° on	80 =	8.0
	2.7 =	.3
41° on	82.7 =	8.3

Therefore, 0955 inch of quickſilver, -	82.7 the approximate height.
Correction for 41° on 82.7 feet,	+8.3
Gives — — —	91.0 = the true height.

Now this was the height of the Tarpeian rock, or the weſt-end of the Capitol-hill in Rome, above the convent of St. Clare, in the *Strada dei ſpecchi*.

The preceding rules for determining heights above the ſurface of the earth will, I preſume, anſwer equally well for meaſuring depths below it.

TABLE I. For the expanſion of quickſilver by heat, ſee p. 62.

Degr. of the Therm.	Height of the barometer in inches.												
	20	21	22	23	24	25	26	27	28	29	30	31	32
1	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2
2	4.1	4.3	4.5	4.7	4.9	5.1	5.3	5.5	5.7	5.9	6.1	6.3	6.5
3	6.1	6.4	6.7	7.0	7.3	7.6	7.9	8.2	8.5	8.8	9.1	9.4	9.7
4	8.1	8.5	8.9	9.3	9.7	10.1	10.5	11.0	11.4	11.8	12.2	12.6	13.0
5	10.1	10.6	11.1	11.6	12.1	12.7	13.2	13.7	14.2	14.7	15.2	15.7	16.2
6	12.2	12.8	13.4	14.0	14.6	15.2	15.8	16.4	17.0	17.6	18.2	18.8	19.5
7	14.2	14.9	15.6	16.3	17.0	17.7	18.4	19.2	19.8	20.6	21.3	22.0	22.7
8	16.2	17.0	17.8	18.6	19.4	20.2	21.0	21.9	22.7	23.5	24.3	25.2	25.9
9	18.2	19.2	20.1	21.0	21.9	22.8	23.7	24.6	25.6	26.5	27.4	28.3	29.2
10	20.3	21.3	22.3	23.3	24.3	25.3	26.3	27.4	28.4	29.4	30.4	31.4	32.4
11	22.3	23.4	24.5	25.6	26.7	27.8	28.9	30.1	31.2	32.3	33.4	34.5	35.6
12	24.3	25.6	26.8	28.0	29.2	30.4	31.6	32.9	34.1	35.3	36.5	37.6	38.9
13	26.3	27.7	29.0	30.3	31.6	32.9	34.2	35.6	36.9	38.2	39.5	40.8	42.1
14	28.4	29.8	31.2	32.6	34.0	35.4	36.8	38.4	39.8	41.2	42.6	43.9	45.4
15	30.4	31.9	33.4	34.9	36.4	37.9	39.4	41.1	42.6	44.1	45.6	47.1	48.6
16	32.4	34.1	35.6	37.2	38.8	40.5	42.0	43.8	45.4	47.0	48.6	50.3	51.8
17	34.5	36.2	37.9	39.6	41.3	43.0	44.7	46.6	48.3	50.0	51.7	53.4	55.1
18	36.5	38.3	40.1	41.9	43.7	45.5	47.3	49.3	51.1	52.9	54.7	56.5	58.3
19	38.5	40.5	42.3	44.2	46.1	48.1	49.9	52.1	54.0	55.9	57.8	59.7	61.6
20	40.6	42.6	44.6	46.6	48.6	50.6	52.6	54.8	56.8	58.8	60.8	62.8	64.9
21	42.6	44.7	46.8	48.9	51.0	53.2	55.2	57.5	59.6	61.7	63.8	65.9	68.1
22	44.6	46.9	49.1	51.3	53.5	55.7	57.9	60.3	62.5	64.7	66.9	69.0	71.4
23	46.6	49.0	51.3	53.6	55.9	58.2	60.5	63.0	65.3	67.6	69.9	72.2	74.6
24	48.6	51.1	53.5	55.9	58.3	60.8	63.1	65.8	68.2	70.6	73.0	75.4	77.8
25	50.7	53.2	55.8	58.2	60.7	63.2	65.7	68.5	71.0	73.5	76.0	78.5	81.1
26	52.7	55.4	58.0	60.5	63.1	65.8	68.3	71.2	73.8	76.4	79.0	81.6	84.3
27	54.7	57.5	60.3	62.9	65.6	68.3	71.0	74.0	76.7	79.4	82.1	84.8	87.5
28	56.8	59.6	62.5	65.2	68.0	70.8	73.6	76.7	79.5	82.3	85.1	87.9	90.7
29	58.8	61.8	64.7	67.5	70.4	73.3	76.2	79.5	82.4	85.3	88.2	91.1	94.1
30	60.8	63.9	66.9	69.9	72.8	75.9	78.9	82.2	85.2	88.2	91.2	94.1	97.3
31	62.8	66.0	69.1	72.2	75.2	78.4	81.5	84.9	88.0	91.1	94.2	97.4	100.5
32	64.8	68.2	71.4	74.6	77.7	81.0	84.2	87.7	90.9	94.1	97.3	100.5	103.8
33	66.9	70.3	73.6	76.9	80.1	83.5	86.8	90.4	93.7	97.0	100.3	103.6	107.0
34	68.9	72.4	75.8	79.2	82.5	86.1	89.4	93.2	96.6	100.0	103.4	106.7	110.3
35	70.9	74.5	78.0	81.5	84.0	88.6	92.0	95.9	99.4	102.9	106.4	109.9	113.5
36	73.0	76.7	80.2	83.8	86.4	91.1	94.6	98.6	102.2	105.8	109.4	113.1	116.8
37	75.0	78.8	82.5	86.2	88.9	93.6	97.3	101.4	105.1	108.8	112.5	116.2	120.0
38	77.0	80.9	84.7	88.5	91.3	96.2	99.9	104.1	107.9	111.7	115.5	119.3	123.2
39	79.0	83.1	86.9	90.8	93.7	98.7	102.5	106.9	110.8	114.7	118.6	122.5	126.5
40	81.1	85.2	89.2	93.2	97.2	101.2	105.2	109.6	113.6	117.6	121.6	125.6	129.7

TABLE II(f). Giving the approximate height in Engliſh feet, adapted to the temperature 31°24 of FAHRENHEIT's thermometer.

Height of the Barom.	Height.	Diff.	Height of the Barom.	Height.	Diff.	Height of the Barom.	Height.	Diff.
Inch.	Feet.		Inch.	Feet.		Inch.	Feet.	
1.—	90309.0		16.10	19570.4	173.1	16.60	17102.5	157.5
2.—	72247.2	18062	20	19398.4	172.0	70	16946.0	156.5
3.—	61681.8	10565	30	19227.5	170.9	80	16790.4	155.6
4.—	54185 4	7496	40	19057.7	169.8	90	16635.8	154.6
5.—	48370.8	5814	50	18889.1	168.6	17.00	16482.1	153.7
6.—	43619.9	4761	60	18721.5	167.6	10	16329.2	152.9
7.—	39603.1	4017	70	18555.0	166.5	20	16177.3	151.9
8.—	36123.6	3480	80	18389.6	165.4	30	16026.2	151.1
9.—	33054.4	3069	90	18225.2	164.4	40	15876.0	150.2
10.—	30309.0	2745	16.00	18061.8	163 4	50	15726.7	149.3
11.—	27825.4	2484	10	17899.4	162.4	60	15578.2	148.5
12.—	25558.1	2267	20	17738.1	161.3	70	15430.6	147.6
13.—	23472.4	2086	30	17577.7	160.4	80	15283.8	146.8
14.—	21541.3	1931	40	17418.4	159.3	90	15137.8	146.0
15.00	19743.5	1798	50	17260.0	158.4	18.00	14992.6	145.2

(f) This table bears ſome analogy to the tables of logiſtical logarithms, being nothing more than the differences of the logarithms of the height of the barometer from the logarithm of 32 inches multiplied by ſix. I have choſen the logarithm of 32 for my term of compariſon, that being the greateſt probable height that the barometer will ever be ſeen at, even at the bottom of the deepeſt mines. Had I taken the mean height of the quickſilver at the level of the ſea, it is true the numbers in the table would have more truly repreſented the heights in the atmoſphere, correſponding to the given height of the quickſilver; but then, in computing ſmall depths or heights from the ſurface of the ſea, we ſhould have been obliged ſometimes to have changed the ſigns in the operation, which appeared to me leſs convenient. The mean height of the barometer at the level of the ſea, from 132 obſervations in Italy and in England, is 30.04 inches, the heat of the barometer being 55°, and the air 62°; ſo that the term of compariſon in this table, *viz.* 32 inches, correſponds to an imaginary point within the earth at 1647 feet below the ſurface of the ſea.

TABLE II. continued.

Height of the Barom.	Height.	Diff.	Height of the Barom.	Height.	Diff.	Height of the Barom.	Height.	Diff.
Inch.	Feet.		Inch.	Feet.		Inch.	Feet.	
18.10	14848.3	144.3	22.00	9763.6	118.8	25.90	5511.0	100.8
20	14704.7	143.6	10	9645.5	118.1	26.00	5410.4	100.6
30	14561.9	142.8	20	9527.8	117.7	10	5310.6	99.8
40	14419.9	142.0	30	9410.7	117.1	20	5210.9	99.7
50	14278.7	141.2	40	9294.1	116.6	30	5111.6	99.3
60	14138.2	140.5	50	9178.1	116.0	40	5012.8	98.8
70	13998.5	139.7	60	9062.5	115.6	50	4914.2	98.6
80	13859.5	139.0	70	8947.4	115.1	60	4816.1	98 1
90	13721.3	138.2	80	8832.9	114.5	70	4718.3	97 8
19.00	13583.8	137.5	90	8718.9	114.0	80	4620.9	97 4
10	13447.0	136.8	23.00	8605.3	113.6	90	4523.9	97.0
20	13310.9	136.1	10	8492.3	113.0	27.00	4427.2	96.7
30	13175.6	135.3	20	8379.7	112.6	10	4330.8	96.4
40	13041.1	134.5	30	8267.6	112.1	20	4234.9	95.9
50	12906.9	134.2	40	8156.0	111.6	30	4139.2	95.7
60	12773.6	133.3	50	8044.9	111.1	40	4044.0	95.2
70	12641.0	132.6	60	7934.3	110.6	50	3949.0	95.0
80	12509.1	131.9	70	7824.1	110.2	60	3854.5	94.5
90	12337.8	131.3	80	7714.4	109.7	70	3760.2	94.3
20.00	12247.2	130.6	90	7605.1	109.3	80	3666.3	93.9
10	12117.2	130.0	24.00	7496.3	108.8	90	3572.7	93.6
20	11987.9	129.3	10	7388.0	108.3	28.00	3479.5	93.2
30	11859.2	128.7	20	7280.1	107.9	10	3386.6	92.9
40	11731.2	128.0	30	7172.6	107.5	20	3294.0	92.6
50	11603.8	127.4	40	7065.6	107.0	30	3201.8	92.2
60	11477.0	126.8	50	6959.0	106.6	40	3109.9	91.9
70	11350.0	126.2	60	6852.9	106.1	50	3018.3	91.6
80	11225.2	125.6	70	6747.2	105.7	60	2927.0	91.3
90	11100.2	125.0	80	6641.9	105.3	70	2836.1	90.9
21.00	10975.8	124.4	90	6537.0	104.9	80	2745.4	90.7
10	10852.1	123.7	25.00	6432.6	104.4	90	2655.1	90.3
20	10728.8	123.3	10	6328.6	104.0	29.00	2565.1	90.0
30	10606.2	122.6	20	6225.0	103.6	10	2475.4	89.7
40	10484.2	122.0	30	6121.8	103.2	20	2386.0	89.4
50	10362.7	121.5	40	6019.0	102.8	30	2296.9	89.1
60	10241.8	120.9	50	5916.6	102.4	40	2208.2	88.7
70	10121.4	120.4	60	5814.6	102.0	50	2119.7	88.5
80	10001.6	119 8	70	5713.0	101.6	60	2031.5	88.2
90	9882.4	119.2	80	5611.8	101.2	70	1943.6	87.9

TABLE II. continued.

Height of the Barom.	Height.	Diff.	Height of the Barom.	Height.	Diff.	Height of the Barom.	Height.	Diff.
Inch.	Feet.		Inch.	Feet.		Inch.	Feet.	
29.80	1856.0	87.6	30.60	1165.7	85.3	31.40	493.2	83.1
90	1768.7	87.3	70	1080.7	85.0	50	410.4	82.8
30.00	1681.7	87.0	80	996.0	84.7	60	327.8	82.6
10	1595.0	86.7	90	911.5	84.5	70	245.4	82.4
20	1508.6	86.4	31.00	827.3	84.2	80	163.4	82.0
30	1422.4	86.2	10	743.4	83.9	90	81.6	81.0
40	1336.6	85.8	20	659.7	83.7	32.00	00.0	81.6
50	1251.0	85.6	30	576.3	83.4			

TABLE III. Of proportional parts.

Diff.	1	2	3	4	5	6	7	8	9	Diff.	1	2	3	4	5	6	7	8	9
81	8	16	24	32	40	49	57	65	73	106	11	21	32	42	53	64	74	85	95
82	8	16	25	33	41	49	57	66	74	107	11	21	32	43	53	64	75	86	96
83	8	17	25	33	41	50	58	66	75	108	11	22	32	43	54	65	76	86	97
84	8	17	25	34	42	50	59	67	76	109	11	22	33	44	54	65	76	87	98
85	8	17	25	34	42	51	59	68	76	110	11	22	33	44	55	66	77	88	99
86	9	17	26	34	43	52	60	69	77	111	11	22	33	44	55	67	78	89	100
87	9	17	26	35	43	52	61	70	78	112	11	22	34	45	56	67	78	90	101
88	9	18	26	35	44	53	62	70	79	113	11	23	34	45	56	68	79	90	102
89	9	18	27	36	44	53	62	71	80	114	11	23	34	46	57	68	80	91	103
90	9	18	27	36	45	54	63	72	81	115	11	23	34	46	57	69	80	92	103
91	9	18	27	36	45	55	64	73	82	116	12	23	35	46	58	70	81	93	104
92	9	18	28	37	46	55	64	74	83	117	12	23	35	47	58	70	82	94	105
93	9	19	28	37	46	56	65	74	84	118	12	24	35	47	59	71	83	94	106
94	9	19	28	38	47	56	66	75	85	119	12	24	36	48	59	71	83	95	107
95	9	19	28	38	47	57	66	76	85	120	12	24	36	48	60	72	84	96	108
96	10	19	29	38	48	58	67	77	86	121	12	24	36	48	60	73	85	97	109
97	10	19	29	39	48	58	68	78	87	122	12	24	37	49	61	73	85	98	110
98	10	20	29	39	49	59	69	78	88	123	12	25	37	49	61	74	86	98	111
99	10	20	30	40	49	59	69	79	89	124	12	25	37	50	62	74	87	99	112
100	10	20	30	40	50	60	70	80	90	125	12	25	37	50	62	75	87	100	112
101	10	20	30	40	50	61	71	81	91	126	13	25	38	50	63	76	88	101	113
102	10	20	31	41	51	61	71	82	92	127	13	25	38	51	63	76	89	102	114
103	10	21	31	41	51	62	72	82	93	128	13	26	38	51	64	77	90	102	115
104	10	21	31	42	52	62	73	83	94	129	13	26	39	52	64	77	90	103	116
105	10	21	31	42	52	63	73	84	94	130	13	26	39	52	65	78	91	104	117

TABLE IV. For the expanſion of the air, or correction of the approximate ~~uppermoſt~~ height, ſee p. 64.

Deg.	Approximate height in feet.								
°	1000.	2000.	3000.	4000.	5000.	6000.	7000.	8000.	9000.
1	2.4	4.9	7.3	9.7	12.1	14.6	17.0	19.4	21.9
2	4.9	9.7	14.6	19.4	24.3	29.2	34.0	38.9	43.7
3	7.3	14.6	21.9	29.2	36.4	43.7	51.0	58.3	65.6
4	9.7	19.4	29.2	38.9	48.6	58.3	68.0	77.8	87.5
5	12.1	24.3	36.4	48.6	60.7	72.9	85.0	97.2	109.3
6	14.6	29.2	43.7	58.3	72.8	87.5	102.0	116.6	131.2
7	17.0	34.0	51.0	68.0	85.0	102.1	119.0	136.1	153.0
8	19.4	38.9	58.3	77.8	97.1	116.6	136.0	155.5	174.9
9	21.9	43.7	65.6	87.5	109.3	131.2	153.0	175.0	196.8
10	24.3	48.6	72.9	97.2	121.5	145.8	170.1	194.4	218.7
11	26.7	53.5	80.2	106.9	133.6	160.4	187.1	213.8	240.6
12	29.2	58.3	87.5	116.6	145.8	175.0	204.1	233.3	262.4
13	31.6	63.2	94.8	126.4	157.9	189.5	221.1	252.7	284.3
14	34.0	68.0	102.1	136.1	170.1	204.1	238.1	272.2	306.2
15	36.4	72.9	109.3	145.8	182.2	218.7	255.1	291.6	328.0
16	38.8	77.8	116.6	155.5	194.3	233.3	272.1	311.0	349.9
17	41.3	82.6	123.9	165.2	206.5	247.9	289.1	330.5	371.7
18	43.7	87.5	131.2	175.0	218.6	262.4	306.1	349.9	393.6
19	46.1	92.3	138.5	184.7	230.8	277.0	323.1	369.4	415.5
20	48.6	97.2	145.8	194.4	243.0	291.6	340.2	388.8	437.4
21	51.0	102.1	153.1	204.1	255.1	306.2	357.2	408.2	459.3
22	53.5	106.9	160.4	213.8	267.3	320.8	374.2	427.7	481.1
23	55.9	111.8	167.7	223.6	279.4	335.3	391.2	447.1	503.0
24	58.3	116.6	175.0	233.3	291.6	349.9	408.2	466.6	524.9
25	60.7	121.5	182.2	243.0	303.7	364.5	425.2	486.0	546.7

TABLE IV. continued.

Deg.	Approximate height in feet.								
0	1000	2000.	3000.	4000	5000.	6000.	7000.	8000.	9000.
26	63.1	126.4	189.5	252.7	315.8	379.1	442.2	505.4	568.6
27	65.6	131.2	196.8	262.4	328.0	393.7	459.2	524.9	590.4
28	68.0	136.1	204.1	272.2	340.1	408.2	476.2	544.3	612.3
29	70.4	140.9	211.4	281.9	352.3	422.8	493.2	563.8	634.2
30	72.9	145.8	218.7	291.6	364.5	437.4	510.3	583.2	656.1
31	75.3	150.7	226.0	301.3	376.6	452.0	527.3	602.6	678.0
32	77.8	155.5	233.3	311.0	388.8	466.6	544.3	622.1	699.8
33	80.2	160.4	240.6	320.8	400.9	480.1	561.3	641.5	721.7
34	82.6	165.2	247.9	330.5	413.1	495.7	578.3	661.0	743.6
35	85.0	170.1	255.1	340.2	425.2	510.2	595.3	680.4	765.4
36	87.4	175.0	262.4	349.9	437.3	524.8	612.3	699.8	787.3
37	89.9	179.8	269.7	359.6	449.5	539.4	629.3	719.3	809.1
38	92.3	184.7	277.0	369.4	461.6	553.9	646.3	738.7	831.0
39	94.7	189.5	284.3	379.1	473.8	568.5	663.3	758.2	852.9
40	97.2	194.4	291.6	388.8	486.0	583.2	680.4	777.6	874.8
41	99.6	199.3	298.9	398.5	498.1	597.8	697.4	797.0	896.7
42	102.1	204.1	306.2	408.2	510.3	612.4	714.4	816.5	918.5
43	104.5	209.0	313.5	418.0	522.4	626.9	731.4	835.9	940.4
44	106.9	213.8	320.8	427.7	534.6	641.5	748.4	855.4	962.3
45	109.3	218.7	328.0	437.4	546.7	656.1	765.4	874.8	984.1
46	111.7	223.6	335.3	447.1	558.8	670.7	782.4	894.2	1006.0
47	114.2	228.4	342.6	456.8	571.0	685.3	799.4	913.7	1027.8
48	116.6	233.3	349.9	466.6	583.1	699.8	816.4	933.1	1049.7
49	119.0	238.1	357.2	476.3	595.3	714.4	833.4	952.6	1071.6
50	121.5	243.0	364.5	486.0	607.5	729.0	850.5	972.0	1093.5

Table

Table of heights taken by the barometer, &c.

	+ or — the Lake of Geneva. Feet.	Above the Mediterranean. Feet.
The Lake of Geneva, from 18 obſervations,	0	1230 *(g)*
Greateſt depth of the Lake,	— 393	
Cluſe, at the Croix Blanche, firſt-floor, *(h)* 2,	+ 351	1581
Chamouny, ground-floor of the inn near the foot of Mont Blanc, 4	+ 2137	3367
The Montanvert, at the Chateâu, 1	+ 5001	6231
The ſource of the river Arvéron, at the bottom of the Vallée de Glace, 1	+ 2426	3656
Salenche, at the inn, ſecond-floor, 1	+ 664	1941
La Bonne-Ville, a la Ville de Geneve, ſecond-floor, 1	+ 245	1475
Chatlaino, country houſe near Geneva, ground-floor, G	+ 178	
The ball on the higheſt, or ſouth-weſt, tower of St. Peter's church in Geneva, G	+ 249	
St. Joire, in a field at the foot of the Mole, G	+ 671	1901
Summit of the Mole,	+ 4883	6113
Pitton, higheſt point of Mont Saleve, G	3284	4514
The Dole, higheſt ſummit of Mont Jura, G	+ 4293	5523
The Buet, G	+ 8894	10124
Aiguille d'Argentiére, G	+ 12172	13402
Mont Blanc, G	+ 14432	15662
Frangy, at the inn, firſt-floor, below the Lake,	— 166	
Aix, a la Ville de Geneve, firſt-floor, below the Lake,	— 378	
Chambery, au St. Jean Baptiſte, firſt-floor, below the Lake,	— 352	
Aiguebelle, at the inn, firſt-floor, below the Lake,	— 190	
La Chambre, at the inn, firſt-floor, above the Lake,	+ 337	
St. Michael, at the inn, firſt-floor,	+ 1113	2343
Modane, at the inn, firſt-floor,	+ 2220	3450

(g) More correctly 1228 feet, but I have taken it at 1230 in round numbers.

(h) The figures at the end of ſome of the names ſhew the number of obſervations that were made; and the letter G indicates ſuch obſervations to have been geometrical.

Table

Table of heights, &c. continued.

	+ or — the Lake of Geneva. Feet.	Above the Mediterranean. Feet.
Lannebourg, the foot of Mont Cenis, at the inn, firſt-floor,	+ 3178	4408
Mont Cenis, at the Poſt (i), — — —	+ 5031	6261
———— at the Grande Croix, — —	+ 4793	6023
Novaleſe, the foot of Mont Cenis on the ſide of Italy, at the inn, firſt-floor, — —	+ 1511	2741
Boucholin, on the firſt-floor, — —	+ 213	
St. Ambroiſe, on the firſt-floor, below the Lake,	— 40	
Turin, a l'Hôtel d'Angleterre, ſecond-floor, 4	— 289	941
Feliſſano, near Aleſſandria, firſt-floor, 1 —	— 671	
Piacenza, St. Marco, firſt-floor, 1 — —	— 967	263
Parma, au Paon, firſt-floor, 3 — —	— 923	307
Bologna, au Pelerin, firſt-floor, 3 —	— 831	399
Loiano, a little village on the Appenines, between Bologna and Florence, — —		2591
The mountain Raticoſa, the higheſt point of the Appenines the road paſſes over, 1¼ miles beyond Filecaije in going to Covigliaje, — —	+ 1671	2901
Florence, nel Corſo dei Tintori, 50 feet above the Arno, which was 18 feet below the wall of the quay, 3	— 990	+ 240
Piſa, aux Trois Demoiſelles, ſecond-floor, 4 —	— 1228	+ 54½
Leghorn, chez Muſton, ſecond-floor, 2 —	— 1244	+ 38
Siena, aux Trois Rois, ſecond-floor, 2 —	— 164	1066
Redicoffani, at the Poſt, firſt-floor, above the Lake,	+ 1240	2470
———— the top of the tower of the old fortification on the ſummit of the rock, —	1830	3060
Viterbo, aux Trois Rois, firſt-floor, on the Ciminus of the Ancients, — — —	+ 29	1259
Rome, nel Corſo, 61 feet above the Tyber, 7	— 1084	94

(i) The rocks on each ſide the plain, where the poſt-houſe ſtands, are at leaſt 3000 feet higher than this ſituation; and it is from the ſnow on the tops, and through the crevices, that the lake on this plain is formed, which gives riſe to the Dora, and may be called one of the ſources of the Po.

Table of heights, &c. continued.

	Above the River Tyber. Feet.	Above the Mediterranean. Feet.
The Level of the river Tyber, — —		33
The top of the Janiculum, near the Villa Spada, —	260	
Aventine Hill, near the Priory of Malta, —	117	
In the Forum, near the arch of SEVERUS, where the ground is raiſed 23½ feet, — —	34	
Palatine Hill, on the floor of the Imperial palace —	133	
Celian Hill, near the CLAUDIAN aqueduct, —	125	
Bottom of the canal of the CLAUDIAN aqueduct, —	175	
Eſquiline Hill, on the floor of St. M. Major's church,	154	
Capitol Hill, on the Weſt-end of the Tarpeian rock,	118	
In the Strada dei Specchi, in the convent of St. Clare,	27	
On the union of the Viminal and Quirinal Hills, in the Carthuſian's church, DIOCLES. Baths, —	141	
Pincian Hill, in the garden of the Villa Medici, —	165	
Top of the croſs of St. Peter's church, —	502	
The baſe of the obeliſk, in the center of the Periſtyle,	31	
The ſummit of the mountain Soracte, lying about 20½ geog. miles N. of Rome, G — —		2271
The ſummit of Monte Velino, one of the Appenines, covered with ſnow in June, about 46 geog. miles N.W. of Rome, and which is probably the higheſt of the Appenines, G — — —		8397
	+ or — the Lake of Geneve.	
Naples, Caſa Iſolata on the Chiaia, 27½ feet above the ſea, 5 — — —	— 1197	
Mount Veſuvius, mouth of the Crater from whence the fire iſſued in 1776, — —		3938 (k)

Table

(k) Sir WILLIAM HAMILTON informed me, that the height of Veſuvius, as taken by Mr. DE SAUSSURE of Geneva in 1772, with only ~~a~~ one barometer of Mr. DE LUC's conſtruction, and according to his rules, was 3659½ French feet = 3900 Engliſh, which agrees pretty well with mine. But the Padre DELLA TORRE pretends to have found the height of Veſuvius in 1752 (ſee p. 44. of his

Table of heights, &c. continued.

	+ or — the Lake of Geneva.	Above the Mediterranean.
Mount Veſuvius, at the baſe of the cone, — —		2021
Top of the mountain Somma, — —		3738
The ſummit of Mount Ætna, — —		10954 (l)
The following heights are determined from correſponding obſervations by Mr. **MESSIER** at Paris, whoſe barometer is ſuppoſed 108 feet above the ſea.		
Barberino di Valdenſa, between Boggebonri and Tavernelle,		974
Modena, a l'Albergo nuovo, — —		214
Montmelian, at 20 feet above the river, — —		811
Monte Viſo, by an obſervation from Turin, by no means accurate, G — — —		9997
Monte Roſa, as meaſured geometrically by the Father **BECCARIA**, being the ſecond mountain of all the Alps, — — — —		15084
Pont Beauvoiſin, — — — —		705
La tour du Pin, 4 — — —		938
Verpilliére, — — — —		566

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his Hiſtory of this Mountain) = 1677 French feet only, the difference of his barometer at the top and at the level of the ſea being no more than 23¼ French lines = 2.065 Engliſh inches, which was certainly a miſtake of little leſs than 2000 feet in the reſult. The Abbé **NOLLET** in 1749 found the fall of the quickſilver 40 lines = 3.55 inches Engliſh; and, if this obſervation is to be depended upon, the ſummit of this volcano has riſen within theſe 27 years more than 330 feet perpendicular.

(l) I have ventured to compute the height of this celebrated mountain from my own tables, though from an obſervation of Mr. **DE SAUSSURE**'s in 1773, which that gentleman obligingly communicated to me. It will ſerve to ſhew, that this volcano is by no means the higheſt mountain of the old world; and that Veſuvius, placed upon Mount Ætna, would not be equal to the height of Mont Blanc, which latter I take to be the moſt elevated point in Europe, Aſia, or Africa.

The circumference of the viſible horizon on the top of Mount Ætna, allowance being made for refraction, which I eſtimate at 6′, is 1093 Engliſh miles.

Table of heights, &c. continued.

	+ or — the Lake of Geneva.	Above the Mediterranean.
Lyons, at the Hôtel Blanc, 50 feet above the Saône,		449
St. Jean le vieux,		695
Cerdon, near the poſt-houſe at the foot of the rocks,		854
Nantua, 10 feet above the Lake,		1423
Chatillon, at the Logis Neuf,		1629
Colonges,		1626
St. Genis, apparently on a level with the foot of Mont Jura,		1501
Geneva, at 100 feet above the Lake, 5		1268 (m)
Mâcon, at the Parc, 24 feet above the Saône		514
Dijon, à la Cloche, the firſt floor,		710
Mountain of Maraiſelois (n), 4¾ miles beyond Viteaux towards Dijon,		1677
Lucy-le-bois,		645
Auxerre, 50 feet above the river,		283
Sens, at the Poſt,		163
Fontainbleau, at the Grand Cerf, ſecond-floor,		242

(m) From this compariſon with Mr. MESSIER's obſervations at Paris, which makes the Lake of Geneva only 1168 feet above the level of the ſea (whereas from 18 obſervations in Italy, near the ſhore of the Mediterranean, it appears to be 1228, *viz.* +60 feet different) I am inclined to believe, that Mr. MESSIER's place of obſervation is about 50 feet higher than I have ſuppoſed it, *viz.* 160 feet above the ſea inſtead of 108, as deduced from three obſervations only at Boulogne, Calais, and at Dover. If this be allowed, the ſame number of feet muſt be added alſo to all the other heights that are determined by compariſon with Mr. MESSIER's obſervations. I am, however, by no means ſure of this, but leave it to future obſervers.

(n) On one ſide of this mountain is a little ſtream called Amançon, that joins the Yonne and the Seine, and thus goes to the Atlantic; while on the other ſide is found the Ouche, which, uniting with the Saône and the Rhone, runs to the Mediterranean: this part of Burgundy then ſeems to be one of the higheſt in France.

Table of heights, &c. continued.

	+ or — the Seine at Paris.	Above the Mediterranean.
Paris, mean height of the Seine, that is, *quand les eaux ſe trouvent à 13 pieds 9 pouces ſur l'echelle du Pont Royal ſelon M.* DE LA LANDE,		36½
Place of my own obſervations in the Ruë Jacob, ſecond-floor,	+ 57	
Mr. MESSIER's obſervatory, at the Hôtel de Clugny, firſt-floor,	72	
Mr. DE LA LANDE's ditto, at the College Royal, firſt-floor,	101	
Place of Monſ. le Pere COTTES's obſervations at Montmorency, 10 miles North of Paris,	333	
Stone-gallery of the Church on Mont Valerien,	473	
Depth of the cave of the Royal Obſervatory at Paris below the pavement,	98½	
The ſame, according to Mr. DE LA LANDE, by actual meaſurement,	98	
Height of the north tower of the church of Notre Dame above the floor,	220½	
—— by actual meaſurement,	218½	
Chantilly,		119
Clermont,		329
Amiens, Ruë de Noyon, firſt-floor,		147
Abbeville, firſt-floor,		79
	Below the mean height of the Seine.	
Boulogne, mean level of the ſea, from one obſerv. only,	—33.9	
Calais, ditto, from one obſervation,	—38.8	
Dover, ditto, from three obſervations made two years preceding thoſe at Calais and Boulogne,	—36.6	
Mean height of the river (o) Thames at London above the mean height of the river Seine from five direct compariſons with Mr. MESSIER,	+ 6.8	
And conſequently the Thames at London above the ſea,		43
Warwick, mean level of the river Avon,		155
Shuckburgh-houſe, in Warwickſhire,		560

(o) By the mean height of the river Thames is underſtood when the water is 15½ feet below the pavement in the left-hand arcade at Buckingham-ſtairs.